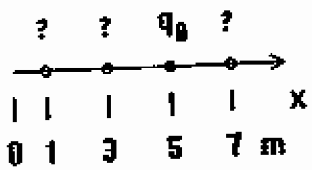
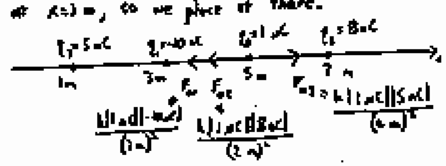
Physics 241 Exam I
Fall 2001

1) A point charge $q_0=1\mu C$ is placed at the position x=5m on the x-axis. Three point charges $q_1=-10nC$, $q_2=8nC$, and $q_3=5nC$ are also placed separately on the x-axis at the three positions x=1m, x=3m, and x=7m WITHOUT SPECIFYING WHICH POSITION WOULD CORRESPOND TO WHICH OF THE THREE CHARGES. Arrange the three charges to find the maximum force on the point charge, q_0, to point in the NEGATIVE-x-direction. The resultant force is ($\hat{i}$ denotes the unit vector in the positive x-direction): [10 points.]



(1) $-12.4\ \mu N\ \hat{i}$
(2) $+11.3\ \mu N\ \hat{i}$
(3) $-5.04\ mN\ \hat{k}$
(4) $-37.7\ \mu N\ \hat{i}$
(5) $-38.5\ \mu N\ \hat{j}$
(6) $+4.35\ mN\ \hat{k}$
(7) 0 N
(8) $-36.5\ \mu N\ \hat{i} + 11.3\ \mu N\ \hat{k}$
(9) $-67.5\ \mu N\ \hat{i}$
(10) $+433\ \mu N\ \hat{i}$

The magnitude of the force exerted on q_0 by a charge q is proportional to $\frac{|q_0||q|}{r^2}$ where r = the separation between q and q_0. We thus consider the charges one at a time, in decreasing order of their magnitude, placing them so as to maximize the force in the $-x$ direction.
We first consider $q_1 = -10\,nC$, as it has the largest magnitude. We have to place it at either $x=1m$ or $x=3m$, so as to produce a force in the $-x$ direction. We place it at $x=3m$, as this minimizes the separation between it and q_0 and therefore maximizes the force.
We next consider $q_2 = 8\,nC$. There are two positions available, at $x=1m$ or $x=7m$. We place it at $x=7m$, as placing it at $x=1m$ would produce a force in the $+x$ direction.
We next consider $q_3 = 5nC$. There is only one space available, at $x=1m$, so we place it there.



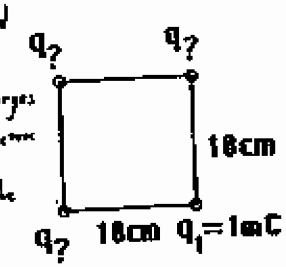
$$\frac{k|1\mu C||5nC|}{(4m)^2} \quad \frac{k|1\mu C||10nC|}{(2m)^2} \quad \frac{k|1\mu C||8nC|}{(2m)^2}$$

$$F_{0x} = F_{03} - F_{01} - F_{02} = k|1\mu C|\left(\frac{5nC}{16} - \frac{10nC}{4} - \frac{8nC}{4}\right) = (8.99\times10^9\ \tfrac{Nm^2}{C^2})(1\times10^{-6}C)\left(\tfrac{5}{16} - \tfrac{10}{4} - \tfrac{8}{4}\right)\times10^{-9}C$$

$$= -37.65\times10^{-6}\ N$$

$\Rightarrow \boxed{\vec{F} = -37.65\ \mu N\ \hat{i}} \Rightarrow \boxed{(4)}$

2) Four charges: $q_1=1mC$, $q_2=-10mC$, $q_3=5mC$, and $q_4=-1mC$ sit on the four corners of a square of side 10cm in the x-y plane. Arrange the charges to yield the maximum (highest possible) total potential energy for the system. What is the resultant potential energy? (Note the energy is taken to be zero when two charges are infinitely separated and ?=2,3, or 4 and no charge occupies more than one corner.) [10 points.]



The electric potential energy is the work required to bring the charges in one at a time and place them at their final positions, assuming the charges are initially infinitely separated. The formula for electric potential energy is $U = \sum_{i=1}^{n} \sum_{j>i} \frac{k q_i q_j}{r_{ij}}$. We bring the charges in from infinity one at a time, in decreasing order of their magnitude.

(1) $-4.99E6$ J
(2) $-2.44E4$ J
(3) $-15.9\ \mu J$
(4) $-48.7\ \mu J$
(5) $-3.25E6$ J
(6) $-4.19E6$ J
(7) $+4.19E6$ J
(8) -3.5 mJ
(9) -1.39 pJ
(10) $+3.0E6$ J

First bring in $q_2 = -10\,mC$. There are no other charges to create $\vec{E}$-fields which we have to do work against when bringing in q_2, so we can bring in q_2 and place it at one corner without doing any work.
We next bring in $q_3 = 5\,mC$. The work to do this is
$U_{23} = \frac{k q_2 q_3}{r_{23}} = \frac{k(-10\,mC)(5\,mC)}{r_{23}} < 0$
The contribution to the electric potential energy is < 0, so we want to make r_{23} as large as possible; we thus place it at the corner opposite to q_2. We have

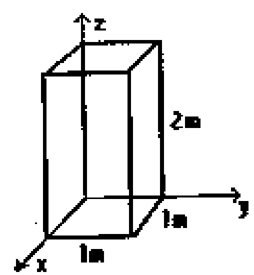
By symmetry, the same U results regardless of whether q_1 is placed at the upper left corner and q_4 at the lower right corner, or vice versa.

$$U = \sum_{i=1}^{4}\sum_{j>i} \frac{k q_i q_j}{r_{ij}} = \frac{k q_1 q_2}{r_{12}} + \frac{k q_1 q_3}{r_{13}} + \frac{k q_1 q_4}{r_{14}} + \frac{k q_2 q_3}{r_{23}} + \frac{k q_2 q_4}{r_{24}} + \frac{k q_3 q_4}{r_{34}}$$

$$= k\left(\frac{(1)(-10)}{a} + \frac{(1)(5)}{a} + \frac{(1)(-1)}{\sqrt{2}a} + \frac{(-10)(5)}{\sqrt{2}a} + \frac{(-10)(-1)}{a} + \frac{(5)(-1)}{a}\right)(mC)^2 = \frac{k}{a}\left(-10+5-\frac{1}{\sqrt{2}}-\frac{50}{\sqrt{2}}+10-5\right)(mC)^2$$

$$= \frac{8.99\times10^9\ \frac{Nm^2}{C^2}}{.1\,m}\left(-\frac{51}{\sqrt{2}}\right)\times10^{-6}C^2 = -3.24\times10^6\ J \Rightarrow \boxed{(5)}$$

3) A closed, Gaussian surface consisting of the six surfaces of a rectangular box with one corner at the origin is drawn in the figure. The box has a square base of sides 1m in length and is 2m tall. Three point charges are placed at three distinct positions as follows: q_1=5nC at (x,y,z) coordinates (0.2m,0.75m,0.9m), q_2=14.6nC at (1.35m,0.2m,1.4m), and q_3=24nC at (0.5m,0.5m,4m). Find the total electric flux through the Gaussian surface. [10 points.]



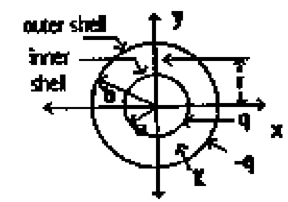
(1) 2.22E3 Vm
(2) 4.93E3 Vm
(3) 3.28E3 Vm
(4) 4.37E3 Vm
(5) 3.73E2 Vm
(6) 6.5E-8 Vm
(7) 4.86E-7 Vm
(8) 3.64E-7 Vm
(9) 5.65E2 Vm
(10) 5.48E-7 Vm

Of the 3 charges, only charge $q_1 = 5\,nC$ is inside the Gaussian surface, so by Gauss' Law

$$\underbrace{\oint_{\text{surface}} \vec{E}\cdot d\vec{A}}_{\text{(electric flux leaving surface)}} = \frac{\text{charge inside surface}}{\epsilon_0} = \frac{q_1}{\epsilon_0} = \frac{5\times10^{-9}\,C}{8.85\times10^{-12}\,\frac{C^2}{Nm^2}} = 564.97\,\frac{Nm^2}{C} = 564.97\,Vm$$

$\Rightarrow \boxed{(9)}$

4) A capacitor is made up of two very long, concentric cylindrical conductors. The inner conductor has a radius a=1mm, while the outer conductor has a radius of b=2mm. A dielectric material with dielectric constant κ=10 fills the volume between the shells. Part I—What is the capacitance for 1 meter of length? [5 points.]



(1) 80.2 pF
(2) 559 pF
(3) 1.12 nF
(4) 55.9 pF
(5) 112 pF
(6) 802 pF
(7) 45 nF
(8) 37.3 mF
(9) 437 mF
(10) 812 nF

Capacitance of cylindrical capacitor: $C = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$

If we use dielectric of dielectric constant κ, new capacitance is

$$C' = \kappa C = \frac{2\pi\kappa\epsilon_0 L}{\ln(b/a)} = \frac{2\pi(10)(8.85\times10^{-12}\frac{C^2}{N\cdot m^2})(1\,m)}{\ln\left(\frac{2\,mm}{1\,mm}\right)}$$
$$= 8.02\times10^{-10}\,F = 802\times10^{-12}\,F = 802\,pF$$

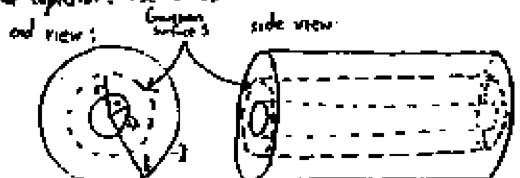
$\Rightarrow \boxed{(6)}$

5) Second part to Problem 4: The dielectric is now removed. A charge per unit length of +λ=1C/m is placed on the inner conductor and −λ=−1C/m on the outer conductor. What is the energy density of the electric field at a point on the y axis a distance r=1.5mm from the center. [5 points.]

(1) 6.4E16 J/m³
(2) 6.4E15 J/m³
(3) 1.6E14 J/m³
(4) 4.0E13 J/m³
(5) 9.4E15 J/m³
(6) 1.6E16 J/m³
(7) 7.7E15 J/m³
(8) 6.4E14 J/m³
(9) 4.0E14 J/m³
(10) 3.8E15 J/m³

The relation between the $\vec{E}$-field at a point and the energy density u at that same point is $u = \frac{1}{2}\epsilon_0|\vec{E}|^2$, so we must find $|\vec{E}|$ at the specified point between the plates of the capacitor. Use Gauss' Law:

end view: Gaussian surface S side view:



$$\oint_S \vec{E}\cdot d\vec{A} = \frac{\text{charge in } S}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$\underbrace{\int_{\text{left end of } S}\vec{E}\cdot d\vec{A}}_{0\ (\vec{E}\perp d\vec{A})} + \underbrace{\int_{\text{right end of } S}\vec{E}\cdot d\vec{A}}_{0\ (\vec{E}\perp d\vec{A})} + \underbrace{\int_{\text{curved part of } S}\vec{E}\cdot d\vec{A}}_{(\vec{E}\parallel d\vec{A})} = |\vec{E}(r)|\underbrace{\boxed{\int_{\text{curved part of } S} dA}}_{2\pi r L}$$

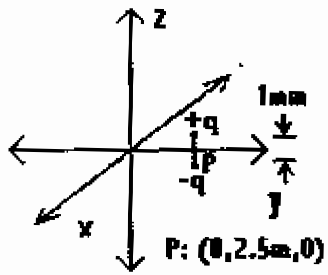
$\Rightarrow |\vec{E}(r)|\cdot 2\pi r L = \frac{\lambda L}{\epsilon_0} \Rightarrow |\vec{E}(r)| = \frac{\lambda}{2\pi\epsilon_0 r}$

$u = \frac{1}{2}\epsilon_0|\vec{E}(r)|^2 = \frac{1}{2}\epsilon_0\frac{\lambda^2}{\epsilon_0^2(2\pi r)^2} = \frac{1}{2\epsilon_0}\left(\frac{\lambda}{2\pi r}\right)^2$

$= \frac{1}{2(8.85\times10^{-12}\frac{C^2}{Nm^2})}\left(\frac{1\,C/m}{2\pi(1.5\times10^{-3}m)}\right)^2 = 6.34\times10^{14}\,\frac{J}{m^3}$

$\Rightarrow \boxed{(8)}$

6) An electric dipole pointing in the positive-z direction consisting of two equal and opposite point charges, q=1C and −q=−1C, spaced 1mm apart sits centered at the point P: (0,2.5m,0) on the y-axis as shown. A uniform electric field of E=[5i + 10j + 20k] N/C is applied. Find the force on the dipole. [5 points.]



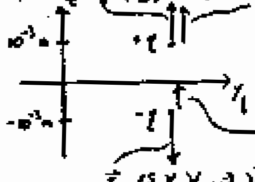
A dipole consists of two charges, equal in magnitude but opposite in sign, joined by a rigid rod of some length d. The net force on a dipole in a uniform E-field is

$\vec{F} = q\vec{E} + (-q)\vec{E} = 0$

$\Rightarrow \boxed{(10)}$

(1) (5i + 10j + 20k) N
(2) (−5i − 10j − 20k) N
(3) (10i + 20j + 40k) N
(4) (−10i − 20j − 40k) N
(5) 5i N
(6) 10i N
(7) 10j N
(8) 20j N
(9) −10i N
(10) 0 N

7) Part II to Problem 6—What is the force on the dipole if the electric field is replaced by E=5(V/m²) z k? [5 points.]



$\vec{E}_+ = (5\frac{V}{m^2})(10^{-3}m)\hat{i}$

$\vec{F}_+ = q\vec{E}_+ = q(5\frac{V}{m^2})(10^{-3}m)\hat{i}$

$\vec{F}_- = (-q)\vec{E}_- = (-q)(5\frac{V}{m^2})(-10^{-3}m)\hat{i}$

$\vec{E}_- = (5\frac{V}{m^2})(-10^{-3}m)\hat{i}$

$\vec{F}_{total} = \vec{F}_+ + \vec{F}_- = 2q(5\frac{V}{m^2})(10^{-3}m)\hat{i}$

$= 2(1C)(5\frac{V}{m^2})(10^{-3}m)\hat{i}$

$= 10^{-2}\,N\hat{i} = 10\times10^{-3}\,N\hat{k} = 10\ mN\ \hat{k}$

$\Rightarrow \boxed{(5)}$

(1) 5.0 mN k
(2) −2.33 pN i
(3) −10 mN i
(4) 3.99 μN i
(5) 10 mN k
(6) 4.35 mN k
(7) 0 N
(8) 5.0 mN j
(9) −1.39 pN i
(10) − 2.52 N j − 5.04 mN k

8) A conductor is 10 meters long and has a uniform rectangular cross section of 3mm by 0.8mm. If the resistivity of the metal making up the conductor is 500μΩ-m what is the resistance? [5 points.]

A = cross-sectional area = $(3\times10^{-3}m)(.8\times10^{-3}m) = 2.4\times10^{-6}\ m^2$

$R = \frac{\rho L}{A} = \frac{(500\times10^{-6}\ \Omega\text{-}m)(10\ m)}{2.4\times10^{-6}\ m^2} = 2083.3\ \Omega$

$\Rightarrow \boxed{(3)}$

(1) 1.2E−10 Ω
(2) 1.2E−9 Ω
(3) 2083 Ω
(4) 1.3E−5 Ω
(5) 1757 Ω
(6) 176 Ω
(7) 208 Ω
(8) 537 Ω
(9) 53.7 Ω
(10) 37 mΩ

9) Part II of 8. If a voltage of 1V is applied across the 10 meter conductor, what is the current density in the wire assuming it is uniform? [5 points.]

A = cross-sectional area of wire = $2.4\times10^{-6}\ m^2$ (from above)

Current density $\vec{J}$ is uniform $\Rightarrow$

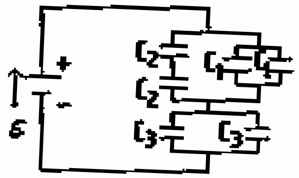
$|\vec{J}| = \frac{i}{A} = \frac{V/R}{A} = \frac{V}{RA} = \frac{1V}{(2083.3\ \Omega)(2.4\times10^{-6}\ m^2)}$

$= 200\ \frac{A}{m^2}$

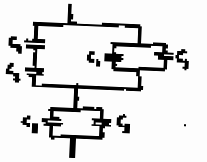
$\Rightarrow \boxed{(10)}$

(1) 100A/m²
(2) 4.8E−4 A/m²
(3) 1.1E−9 A/m²
(4) 4.8E−3 A/m²
(5) 6.1E−2 A/m²
(6) 1.6E−2 A/m²
(7) 4.8E−6 A/m²
(8) 50 A/m²
(9) 133A/m²
(10) 200A/m²

10) For the circuit in the diagram C_1=5nF, C_3=10nF, and the equivalent capacitance, C_{eq} =10nF for the entire circuit. Find C_2, for the circuit. [10 points.]



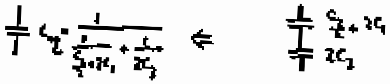
(1) 5 nF
(2) 20 nF
(3) 8 nF
(4) 15 nF
(5) 35 nF
(6) 33.5 nF
(7) 10 nF
(8) 430 nF
(9) 2 nF
(10) 1 nF



$\Rightarrow \frac{C_2}{2} = \frac{1}{\frac{1}{C_2}+\frac{1}{C_2}}$ $\quad C_1+C_1=2C_1$ $\quad C_3+C_3=2C_3$

$\Downarrow$

$\frac{C_2}{2}+2C_1$, $2C_3$



$C_{eq} = \frac{1}{\frac{1}{\frac{C_2}{2}+2C_1}+\frac{1}{2C_3}} \Leftarrow$

$\frac{1}{C_{eq}} = \frac{1}{\frac{C_2}{2}+2C_1}+\frac{1}{2C_3} \Rightarrow \frac{1}{\frac{C_2}{2}+2C_1} = \frac{1}{C_{eq}} - \frac{1}{2C_3}$

$\Rightarrow \frac{C_2}{2}+2C_1 = \frac{1}{\frac{1}{C_{eq}}-\frac{1}{2C_3}}$

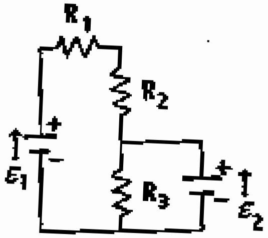
$\Rightarrow C_2 = 2\left(\frac{1}{\frac{1}{C_{eq}}-\frac{1}{2C_3}} - 2C_1\right) = 2\left(\frac{1}{\frac{1}{10\,nF}-\frac{1}{2(10\,nF)}} - 2(5\,nF)\right)$

$= 2\left(\frac{1}{\frac{1}{10}-\frac{1}{20}} - 10\right) nF = 2\left(\frac{1}{\frac{1}{20}} - 10\right) nF = 2(20-10)\,nF$

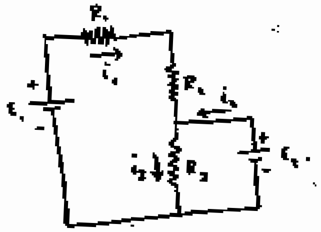
$= 20\,nF$

$\Rightarrow \boxed{(2)}$

11) The following circuit has E_1=100V, E_2=50V, R_1=100Ω, R_2=150Ω. Find R_3 so that no current flows into or out of battery#2 (E_2). [10 points.]



(1) 100 Ω
(2) 60 Ω
(3) 190 Ω
(4) 450 Ω
(5) 150 Ω
(6) 1000Ω
(7) 250 Ω
(8) 500 Ω
(9) 50 Ω
(10) 300 Ω



KVL (outer loop): $\mathcal{E}_1 - i_1R_1 - i_1R_2 - \mathcal{E}_2 = 0$ (1)

KVL (right loop): $\mathcal{E}_2 - i_3R_3 = 0$ (2)

KCL : $i_3 = i_1 + i_2$ (3)

(1) $\Rightarrow$ $i_1 = \frac{\mathcal{E}_1-\mathcal{E}_2}{R_1+R_2}$ (4)

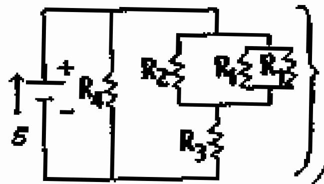
Imposing the constraint $i_2 = 0$, (3) $\Rightarrow i_1 = i_3$ (5)

(2) $\Rightarrow$ $R_3 = \frac{\mathcal{E}_2}{i_3} \underset{\text{by (5)}}{=} \frac{\mathcal{E}_2}{i_1} \underset{\text{by (4)}}{=} \frac{\mathcal{E}_2}{\frac{\mathcal{E}_1-\mathcal{E}_2}{R_1+R_2}} = \frac{\mathcal{E}_2(R_1+R_2)}{\mathcal{E}_1-\mathcal{E}_2}$

$= \frac{(50\,V)(100+150\,\Omega)}{(100-50\,V)} = 250\,\Omega$

$\Rightarrow \boxed{(7)}$

12) Find the voltage across the resistors, R_3, for the circuit in the figure below. $R_1=100\Omega$, $R_2=200\Omega$, $R_3=50\Omega$, $R_4=180\Omega$, and $E=50$Volts. [10 points.]



(1) 24.7 V

(2) 9.3 V

(3) 40.7 V

(4) 27.8 V

(5) 8.8 V

(6) 41.2 V

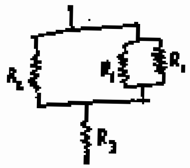
(7) 22.2 V

(8) 50 V

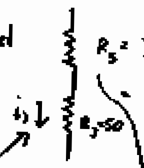
(9) 26.3 V

(10) 0 V

The battery forces the voltage across here to be $\mathcal{E}$, so we don't have to worry about R_4.



R_1, R_1, R_1 in parallel $\Rightarrow$



$R_5 = \frac{1}{\frac{1}{R_1}+\frac{1}{R_1}+\frac{1}{R_1}} = \frac{1}{\frac{1}{100}+\frac{1}{100}+\frac{1}{200}} = 40$

$\Downarrow$

$50V = \mathcal{E}$ $R = R_3 + R_5 = 40 + 50 = 90$

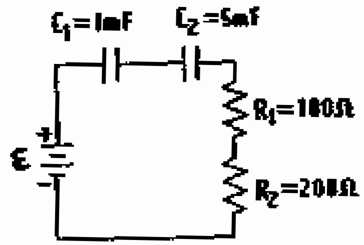
$i = \frac{V}{R} = \frac{50\,V}{90} = .555\ A$

So current here $(i_3) = .555\ A$

$\Rightarrow V_{R_3} = i_3 R_3 = (.555\ A)(50\ \Omega) = 27.77\ V$

$\Rightarrow \boxed{(4)}$

13) What is the time constant for charging capacitor C_2 in the circuit shown? [10 points.]



(1) 1.8 s

(2) 1.2 s

(3) 1.5 s

(4) 0.5 s

(5) 0.25 s

(6) 0.1 s

(7) 1 s

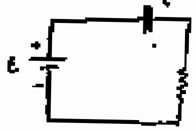
(8) 0.2 s

(9) 0.17 s

(10) 83 ms

$\Downarrow$

$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{1}{\frac{1}{1mF}+\frac{1}{5mF}} = .833\ mF$



$R_{eq} = R_1 + R_2 = 100 + 200 = 300\ \Omega$

Capacitive time constant is

$\tau = R_{eq} C_{eq} = (300\ \Omega)(.833 \times 10^{-3}\ F) = .25\ s$

$\Rightarrow \boxed{(5)}$