Exam 1
October 2, 2003

Physics 241

1. Please print your name on the top edge of the op-scan sheet.
2. Use a #2 pencil to fill in your full name, your student identification number, your recitation division number, and finally the answers for problems 1–12.
3. One (both sides) 8 1/2" x 11" crib sheet is allowed. It must be hand-written.

Useful equations and constants:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \vec{E} = \vec{F}/q_0 \qquad dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$\vec{\tau} = \vec{p}\times\vec{E} \qquad \phi_E = \oint \vec{E}\cdot d\vec{A} \qquad \varepsilon_0\phi_E = q_{enclosed}$$

$$V_b - V_a = \frac{W_{ab}}{q_0} = -\int_a^b \vec{E}\cdot d\vec{l} \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$U = Vq \qquad E = -\frac{dV}{dl} \qquad q = CV \qquad C = \varepsilon_0\frac{A}{d} \qquad C = \kappa C_0$$

$$R = \rho\frac{L}{A} \qquad V = iR \qquad P = iV \qquad U = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}\frac{q^2}{C} \qquad V = \varepsilon(1 - e^{-t/RC}) \qquad i = \frac{\varepsilon}{R}e^{-t/RC} \qquad q = q_0 e^{-t/RC}$$

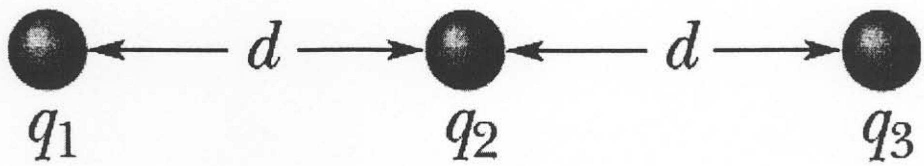
$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\frac{N\cdot m^2}{C^2} \qquad \varepsilon_0 = 8.85\times10^{-12}\frac{C^2}{N\cdot m^2}$$

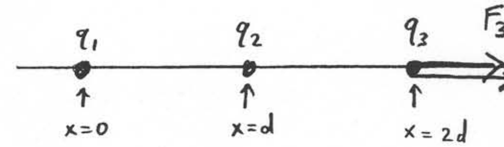
$$e = 1.6\times10^{-19}\,\mathrm{C} \qquad m_p = 1.67\times10^{-27}\,kg$$

$$\mu \Rightarrow 10^{-6}\; n \Rightarrow 10^{-9}\; p \Rightarrow 10^{-12}$$

$$\text{For } ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. In the figure below, three charged particles lie in a straight line and are separated by a distance d. Charges q_1 and q_2 are held fixed. Charge q_3 is free to move but happens to be in equilibrium (no electrostatic force acts on it).If q_2 = 1.5 μC find the value of q_1.





$$F_{31}=\frac{kq_1q_3}{(2d)^2}$$

$$F_{32}=\frac{kq_2q_3}{d^2}$$

(a) -3.0 μC
(b) -6.0 μC
(c) +3.0 μC
(d) +6.0 μC
(e) none of the above

$$F_{3x}=F_{31}+F_{32}=\frac{kq_1q_3}{(2d)^2}+\frac{kq_2q_3}{d^2}$$

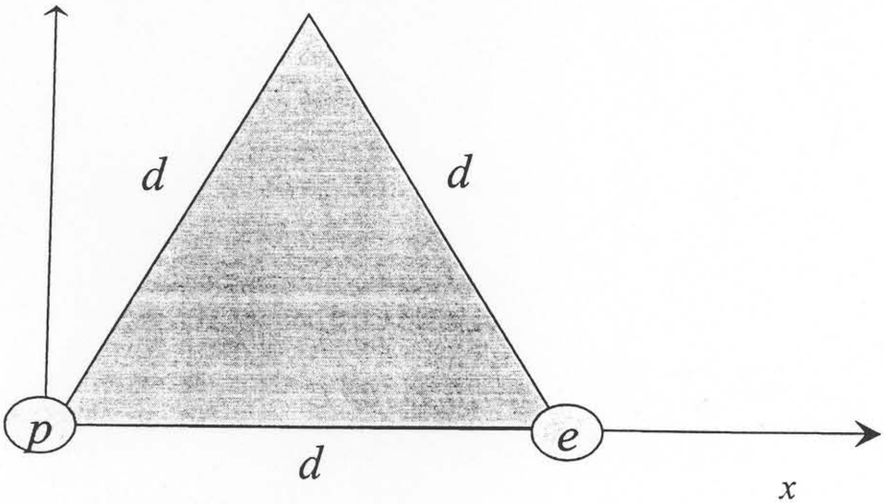
Set this $=0$

$$\Rightarrow\ 0=\frac{kq_1q_3}{4d^2}+\frac{kq_2q_3}{d^2}$$

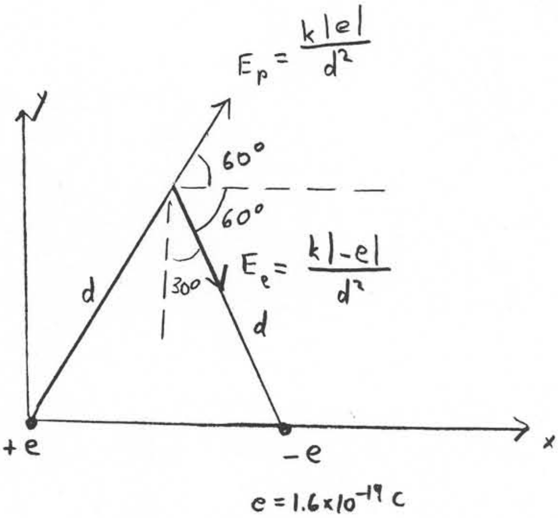
$$\Rightarrow\ \frac{kq_1q_3}{4d^2}=-\frac{kq_2q_3}{d^2}\ \Rightarrow\ \frac{q_1}{4}=-q_2$$

$$\Rightarrow\ q_1=-4q_2=-4(1.5\,\mu C)=\boxed{-6\,\mu C}$$

2. A proton and an electron form the two corners of an equilateral triangle of side length $d=4.0\times10^{-6}$ m. Assume that the proton is located at the origin and that the electron is on the positive x-axis. What is the magnitude and direction of their net electric field at the third corner?



(a) 90 N/C, perpendicular to the x-axis
(b) 45 N/C, 60 degrees with the x-axis
(c) 45 N/C, parallel to the x-axis
(d) 180 N/C, parallel to the x-axis
(e) 90 N/C, parallel to the x-axis

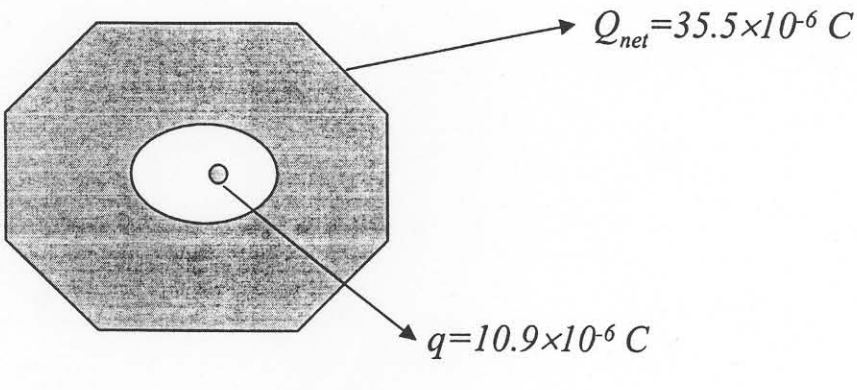


$$E_x = E_p \cos 60^\circ + E_e \cos 60^\circ$$
$$= \left(\frac{k|e|}{d^2}\right)\cos 60^\circ + \left(\frac{k|-e|}{d^2}\right)\cos 60^\circ$$
$$= 2\,\frac{k|e|}{d^2}\underbrace{\cos 60^\circ}_{\frac{1}{2}} = \frac{k|e|}{d^2}$$
$$= \frac{(8.99\times10^9\ \frac{N\,m^2}{C^2})(1.6\times10^{-19}\,C)}{(4.0\times10^{-6}\,m)^2} = 90\ \frac{N}{C}$$

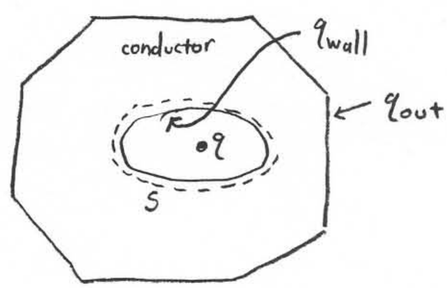
$$E_y = E_p \sin 60^\circ - E_e \sin 60^\circ$$
$$= \left(\frac{k|e|}{d^2}\right)\sin 60^\circ - \left(\frac{k|-e|}{d^2}\right)\sin 60^\circ = 0$$

$$\boxed{|\vec{E}| = 90\ \frac{N}{C},\ \vec{E} \text{ points in } +x \text{ direction}}$$

3. An isolated conductor of arbitrary shape has a net charge Q_{net}=+35.5 ×10^{-6} C. Inside the conductor there is a cavity within which is a point charge q = 10.9 x10^{-6} C. What is the charge q_{wall} on the cavity wall and the charge q_{out} on the outer surface of the conductor?



(a) q_{wall}= -46.4 ×10^{-6} C q_{out}= 46.4 ×10^{-6} C
(b) q_{wall}= 0 ×10^{-6} C q_{out}= -35.5 ×10^{-6} C
(c) q_{wall}= -10.9 ×10^{-6} C q_{out}= 46.4 ×10^{-6} C
(d) q_{wall}= -10.9 ×10^{-6} C q_{out}= 35.5 ×10^{-6} C
(e) q_{wall}= 0 ×10^{-6} C q_{out}= 35.5 ×10^{-6} C



Apply Gauss' Law to Gaussian surface S:

$$\underbrace{\oint_S \vec{E}\cdot d\vec{A}}_{=0} = \frac{\text{charge in } S}{\epsilon_0} = \frac{(q+q_{wall})}{\epsilon_0} \Rightarrow 0 = q + q_{wall}$$

since surface S is inside the conductor, and $\vec{E}=0$ there

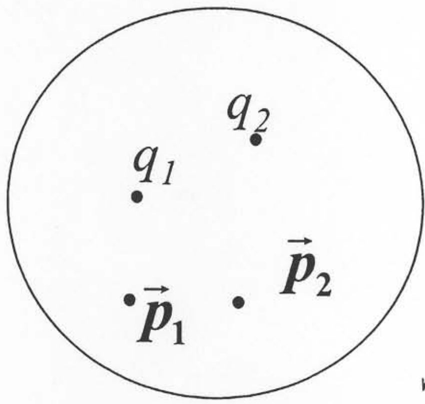
$$\Rightarrow \boxed{q_{wall} = -q = -10.9\times10^{-6}\ C}$$

$$Q_{net} = q_{wall} + q_{out}$$

$$\Rightarrow q_{out} = Q_{net} - q_{wall} = (35.5\times10^{-6}\ C) - (-10.9\times10^{-6}\ C)$$

$$= \boxed{46.4\times10^{-6}\ C}$$

4. Consider a spherical Gaussian surface of radius 1 m which surrounds two electric dipoles and two charges (one positive and one negative) as shown below. Here $q_1 =$ 7 nC, $q_2 = -4$ nC, and $p_1 = p_2 = 10^{-10}$ C m. What is the net electric flux through the Gaussian surface?



$\vec{p}_1, \vec{p}_2$ represent the electric dipole moment vectors of the two dipoles.

(a) 561 Nm^2/C
(b) 7059 Nm^2/C
(c) 4649 Nm^2/C
(d) 339 Nm^2/C
(e) 2325 Nm^2/C

net charge in Gaussian surface $S = q_1 + q_2$

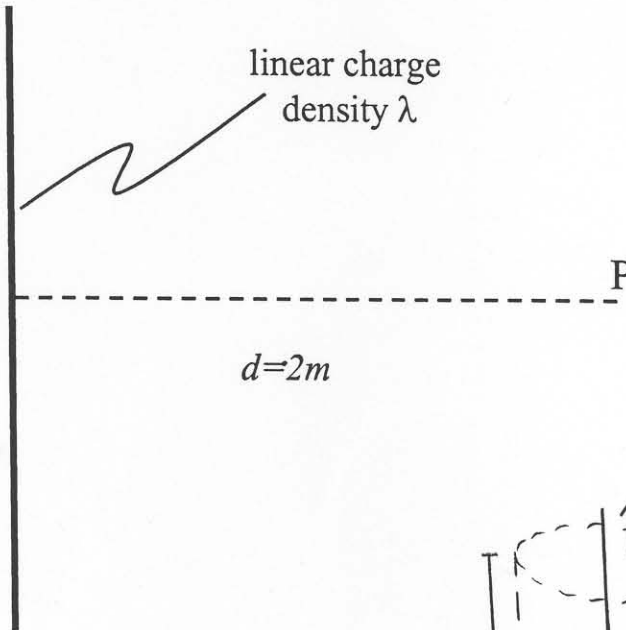
(a dipole consists of two charges, equal in magnitude and opposite in sign, so the dipoles don't contribute anything to the net charge)

By Gauss' law, $\underbrace{\oint_S \vec{E} \cdot d\vec{A}}_{\Phi_E} = \frac{\text{charge in } S}{\epsilon_0} = \frac{q_1 + q_2}{\epsilon_0}$

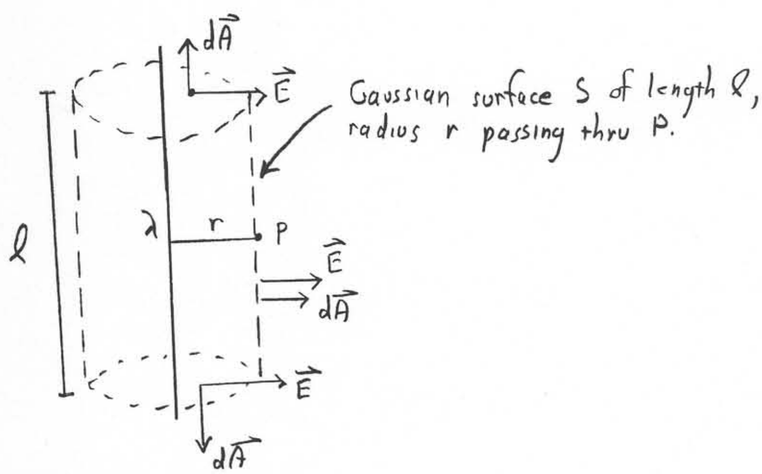
Φ_E (Electric flux leaving the surface S)

$$\Rightarrow \Phi_E = \frac{q_1 + q_2}{\epsilon_0} = \frac{(7 \times 10^{-9}\,C) + (-4 \times 10^{-9}\,C)}{8.85 \times 10^{-12} \frac{C^2}{Nm^2}} = \boxed{339 \frac{Nm^2}{C}}$$

5. An infinite line of charge produces a field E= 4×10^{6} N/C at a point P that is a distance of 2 m from the line. Calculate the linear charge density λ.



(a) 4.45×10^{-4} C/m
(b) 8 C/m
(c) 0.89×10^{-3} C/m
(d) 1.7×10^{-3} C/m
(e) 8.94 C/m



Apply Gauss' law: $\underbrace{\oint_S \vec{E}\cdot d\vec{A}}_{\parallel} = \dfrac{\text{charge in } S}{\epsilon_0} = \dfrac{\lambda \ell}{\epsilon_0}$

$$\underbrace{\int_{\substack{\text{top}\\ \text{end}}} \vec{E}\cdot d\vec{A}}_{\substack{\parallel \\ 0 \\ \text{since } \vec{E}\perp d\vec{A} \\ \text{on top}}} + \underbrace{\int_{\substack{\text{bottom}\\ \text{end}}} \vec{E}\cdot d\vec{A}}_{\substack{\parallel \\ 0 \\ \text{since } \vec{E}\perp d\vec{A} \\ \text{on bottom}}} + \int_{\substack{\text{curved part}\\ \text{of } S}} \underbrace{\vec{E}\cdot d\vec{A}}_{\substack{\parallel \\ |\vec{E}|\,|d\vec{A}| \\ \parallel \\ dA}} = |\vec{E}(r)| \overbrace{\boxed{\int_{\substack{\text{curved}\\ \text{part of } S}} dA}}^{\substack{2\pi r \ell \\ \parallel}}$$

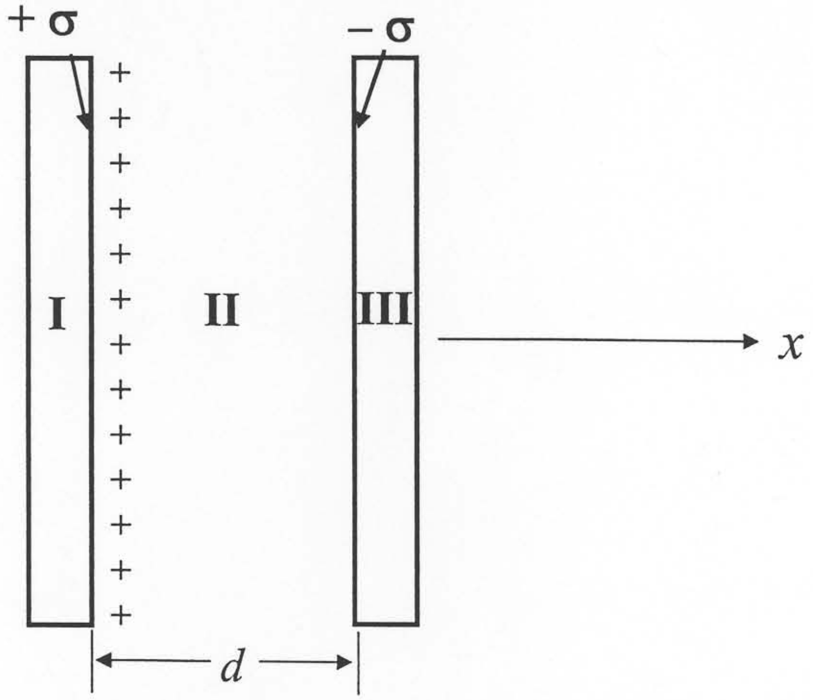
since $\vec{E} \parallel d\vec{A}$ on curved part

$|\vec{E}|$ doesn't vary over curved part of S

$$\Rightarrow |\vec{E}(r)|\, 2\pi r \ell = \frac{\lambda \ell}{\epsilon_0} \Rightarrow |\vec{E}(r)| = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Rightarrow \lambda = 2\pi\epsilon_0 r\, |\vec{E}(r)| = 2\pi\left(8.85\times10^{-12}\ \frac{C^2}{Nm^2}\right)(2\,m)\left(4\times10^{6}\ \frac{N}{C}\right) = \boxed{4.45\times10^{-4}\ \frac{C}{m}}$$

6. Consider two infinite metal plates a distance d apart.

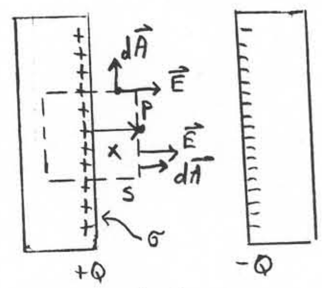


The plate on the left carries a uniform surface charge density of $+\sigma$, while the plate on the right carries a uniform surface charge density of $-\sigma$. What is the x-component of the electric field in regions I and II?

	region I	region II
(a)	0	$\frac{\sigma}{2\varepsilon_0}$
(b)	0	$\frac{\sigma}{\varepsilon_0}$
(c)	0	$\frac{2\sigma}{\varepsilon_0}$
(d)	$\frac{\sigma}{\varepsilon_0}$	0
(e)	None of the above	

region I is inside the conductor, so $\vec{E}=0$ there.

region II: Consider a point P in region II a distance x from the + plate



Let S be a cylindrical Gaussian surface of cross-sectional area A' passing thru the point P.

Gauss' Law: $\underbrace{\oint_S \vec{E}\cdot d\vec{A}}_{\parallel} = \frac{\text{charge in } S}{\epsilon_0} = \frac{\sigma A'}{\epsilon_0}$

$$\underbrace{\int_{\text{left end}} \vec{E}\cdot d\vec{A}}_{\substack{\parallel \\ 0 \\ \text{since } \vec{E}=0 \text{ in} \\ \text{conductor}}} + \underbrace{\int_{\text{right end}} \vec{E}\cdot d\vec{A}}_{|\vec{E}|\,dA} + \underbrace{\int_{\text{sides}} \vec{E}\cdot d\vec{A}}_{\substack{\parallel \\ 0 \\ \text{since } \vec{E}\perp d\vec{A} \\ \text{over sides}}}$$

$$\parallel$$

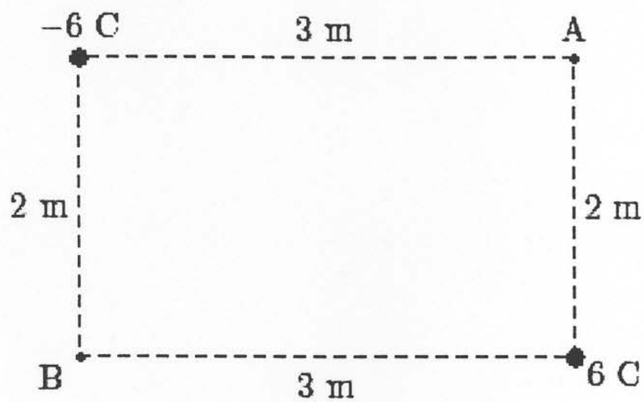
$$|\vec{E}(x)|\boxed{\int_{\text{right end}} dA} = A'$$

$\Rightarrow |\vec{E}(x)|A' = \frac{\sigma A'}{\epsilon_0}$

$\Rightarrow \boxed{\begin{array}{ll} |\vec{E}(x)| = \frac{\sigma}{\epsilon_0} & (\text{region II}) \\ \vec{E}(x) = 0 & (\text{region I}) \end{array}}$

13 pages total

7. Find the difference in potential between the points A and B (i.e. find V(B) - V(A)). Assume that the potential is zero at infinity.



(a) 0 V
(b) 8.99×10^9 V
(c) -8.99×10^9 V
(d) 1.8×10^{10} V
(e) -1.8×10^{10} V

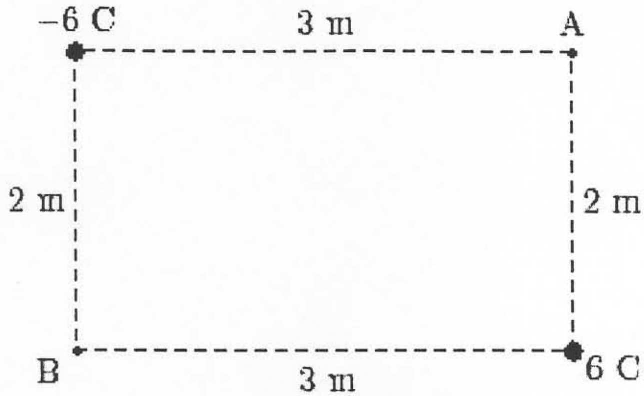
$$V(A) = \frac{k(6C)}{2m} + \frac{k(-6C)}{3m} = 3k - 2k = k$$

$$V(B) = \frac{k(6C)}{3m} + \frac{k(-6C)}{2m} = 2k - 3k = -k$$

$$V(B) - V(A) = -k - (k) = -2k = -2(8.99 \times 10^9)$$

$$= \boxed{-1.8 \times 10^{10}\ V}$$

8. In the previous problem, what would be the work done by an external agent in moving a 4 C charge from the point A to the point B? (Assume that the potential is zero at infinity).



(a) 0 J
(b) -7.2 × 10^{10} J
(c) 7.2 × 10^{10} J
(d) 1.8 × 10^{10} J
(e) -1.8 ×10^{10} J

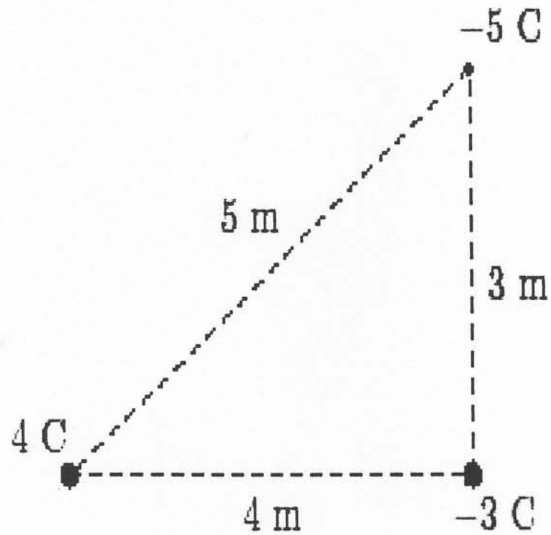
From previous problem $V(B) - V(A) = -1.8 \times 10^{10}$ V

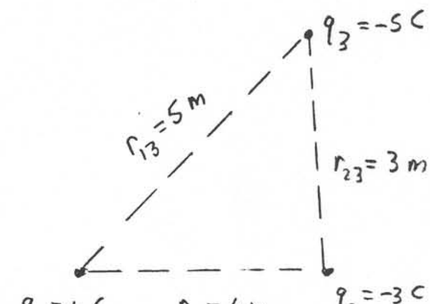
Work done by external agent in moving a 4C charge from A → B is

$$W^{ext}_{A \to B} = (4C)\,(V(B) - V(A))$$

$$= (4C)(-1.8 \times 10^{10}\ V) = \boxed{-7.2 \times 10^{10}\ J}$$

9. Find the Electric Potential Energy of the configuration of point charges below. (Assume that the potential is zero at infinity).





$$U = \sum_{i=1}^{3} \sum_{\substack{j=1 \\ j>i}}^{3} \frac{k q_i q_j}{r_{ij}} = \frac{k q_1 q_2}{r_{12}} + \frac{k q_1 q_3}{r_{13}} + \frac{k q_2 q_3}{r_{23}}$$

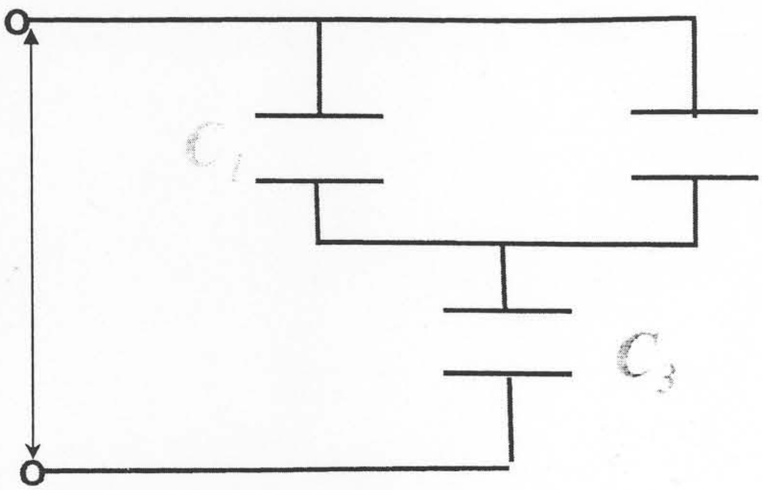
$$= k\left(\frac{(4\,c)(-3\,c)}{4\,m} + \frac{(4\,c)(-5\,c)}{5\,m} + \frac{(-3\,c)(-5\,c)}{3\,m} \right)$$

$$= k(-3 - 4 + 5) = -2k$$

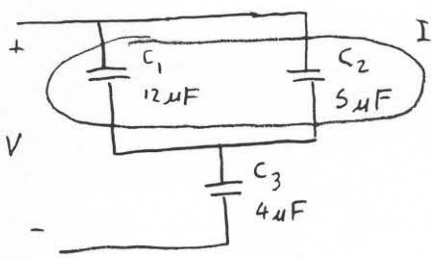
$$= -2(8.99 \times 10^9) = \boxed{-1.8 \times 10^{10}\ J}$$

(a) -1.8×10^{10} J
(b) 3.6×10^9 J
(c) 1.8×10^{10} J
(d) 5.4×10^{10} J
(e) -3.6×10^9 J

10. What is the charge on C_3 if the potential difference applied to the input terminals of the circuit below is V=9V



Assume: C_1=12 μF, C_2=5 μF and C_3=4 μF.

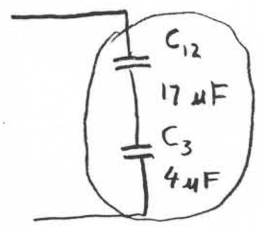


In parallel.

$C_{eq} = C_1 + C_2 = 12 + 5 = 17\ \mu F$

a) 180 μC
b) 45 μC
c) 36 μC
d) 29.1 μC
e) none of the above

$\Rightarrow$

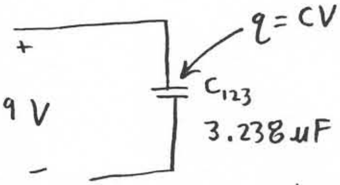


In series

$C_{eq} = \dfrac{1}{\frac{1}{C_{12}} + \frac{1}{C_3}} = \dfrac{1}{\frac{1}{17} + \frac{1}{4}}$

$= 3.238\ \mu F$

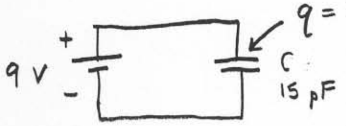
$q = CV = (3.238\ \mu F)(9V) = 29.14\ C$



Capacitors in series have the same charge on each, which is same as charge on equivalent capacitance, so charge on both C_{12} and C_3 is $\boxed{29.1\ \mu C}$

11. A parallel-plate capacitor whose capacitance C is 15 pF is charged by a battery so that the potential difference between the plates of the capacitor is V=9V. The charging battery is now disconnected and a porcelain slab (κ=6.50) is slipped in between the plates. What is the potential energy of the capacitor-slab device after the slab is put into place?

$q = CV = (15 \times 10^{-12}\ F)(9\ V) = 1.35 \times 10^{-10}\ C$



(a) 6.08×10^{-10} J
(b) 3.95×10^{-9} J
(c) 9.35×10^{-11} J
(d) 1.22×10^{-9} J
(e) 1.87×10^{-10} J

After slab put in, new capacitance becomes

$$C_{new} = \kappa C_{old} = (6.5)(15 \times 10^{-12}\ F) = 9.75 \times 10^{-11}\ F$$

Energy stored in new capacitor is

$$U = \frac{q^2}{2C_{new}} = \frac{(1.35 \times 10^{-10}\ C)^2}{2(9.75 \times 10^{-11}\ C)} = \boxed{9.35 \times 10^{-11}\ J}$$

12. Consider a parallel plate capacitor having plates of area A separated by a distance d, where the space between the plates is filled with a dielectric of dielectric constant κ. Assume that this capacitor has a capacitance of C. If I construct a new capacitor by doubling the plate area (so A' = 2A), doubling the plate separation (so d ' = 2d), and doubling the dielectric constant (so κ ' = 2 κ), what is the capacitance C' of this new capacitor in terms of the old capacitance C?

old capacitance: $C = \frac{\kappa A \epsilon_0}{d}$

(a) 4C
(b) 8C
(c) C/2
(d) C/4
(e) 2C

new capacitance: $C' = \frac{\kappa' A' \epsilon_0}{d'} = \frac{(2\kappa)(2A)\epsilon_0}{(2d)}$

$= 2\underbrace{\left(\frac{\kappa A \epsilon_0}{d}\right)}_{C} = \boxed{2C}$

Exam I - Phys 241

Form A		Form B	
1.	B	1.	D
2.	E	2.	E
3.	C.	3.	B
4.	D	4.	A
5.	A	5.	B
6.	B	6.	A
7.	E	7.	C
8.	B	8.	D
9.	A	9.	C.
10.	D	10.	A
11.	C.	11.	E
12.	E	12.	C