Physics 241 Exam 1
September 28, 2004

One (both sides) 8 1/2" x 11" crib sheet is allowed. It must be of your own creation.
Useful equations and constants:

$$F = k\frac{q_1 q_2}{r^2} \qquad \vec{E} = \frac{\vec{F}}{q_o} \qquad dE = k\frac{dq}{r^2}$$

$$\vec{\tau} = \vec{p}\times\vec{E} \qquad \phi_E = \oint \vec{E}\cdot d\vec{A} \qquad \phi_E = \frac{Q_{inside}}{\varepsilon_o}$$

$$V_b - V_a = \frac{\Delta U}{q_0} = -\int_a^b \vec{E}\cdot d\vec{l} \qquad W_{ab} = q\Delta V \qquad V = k\frac{q}{r} \qquad \vec{E} = -\vec{\nabla}V$$

$$Q = CV \qquad U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV \qquad v_f^2 - v_i^2 = 2a\Delta y$$

$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}} \qquad \varepsilon_0 = 8.85\times10^{-12}\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$$

$$e = 1.602\times10^{-19}\mathrm{C} \qquad c = 2.99792458\times10^8 \mathrm{m/s} \text{ (speed of light)}$$

$$N_{Avogadro} = 6.022\times10^{23} \qquad m_p = 1.67\times10^{-27}kg$$

$$\mathrm{m}\Rightarrow10^{-3} \qquad \mu\Rightarrow10^{-6} \qquad \mathrm{n}\Rightarrow10^{-9} \qquad \mathrm{p}\Rightarrow10^{-12} \qquad \mathrm{f}\Rightarrow10^{-15}$$

$$\mathrm{k}\Rightarrow10^{3} \qquad \mathrm{M}\Rightarrow10^{6} \qquad \mathrm{G}\Rightarrow10^{9} \qquad \mathrm{T}\Rightarrow10^{12} \qquad \mathrm{P}\Rightarrow10^{15}$$

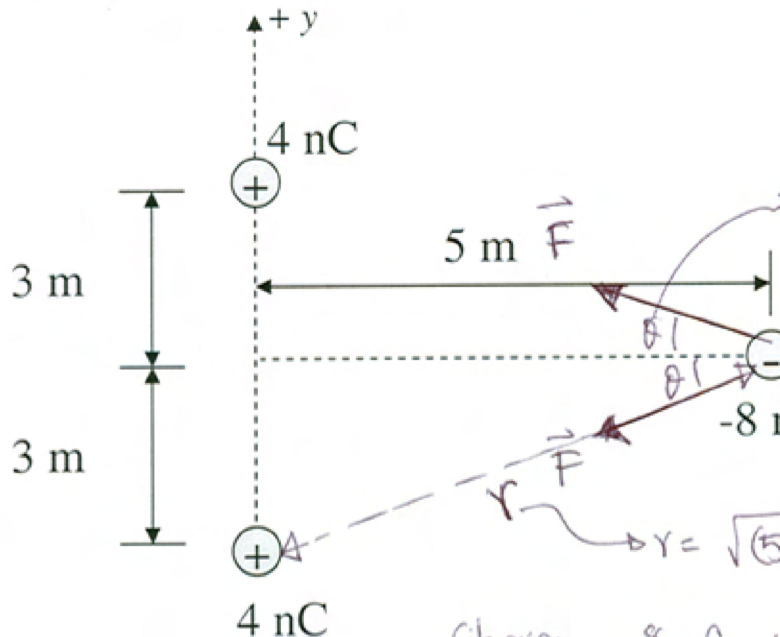
$$\text{For } ax^2 + bx + c = 0$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1. Please sign the opscan sheet and print your name on it.
2. Use a #2 pencil to fill in your full name, your social security number, and finally the answers for problems 1–12.
3. Please be prepared to show your Purdue ID when you hand in your opscan sheet.

blue

1.- Three charges are arranged as shown in the figure below. Find the magnitude and direction of the electrostatic force on the -8 nC charge.



$\theta = \tan^{-1}\left(\frac{3}{5}\right)$

$\theta = 30.96°$

$r = \sqrt{(5m)^2 + (3m)^2} = 5.83m$

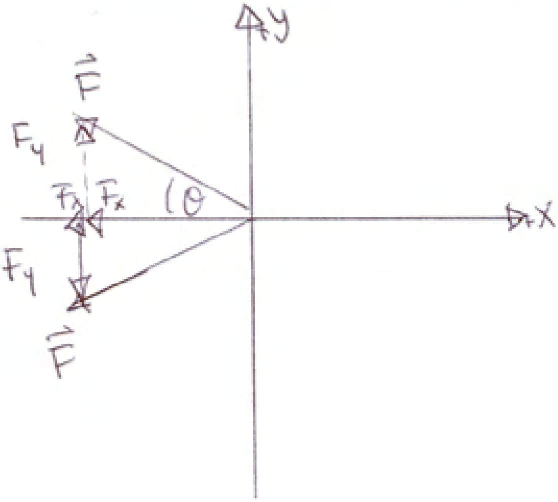
Charge -8nC would be attracted to both +4nC charges. The magnitude is

$$|\vec{F}| = \frac{K|q_1 q_2|}{r^2} =$$

$$|\vec{F}| = \frac{(9\times10^9 \frac{N\cdot C^2}{m^2})(4\times10^{-9}C)(8\times10^{-9}C)}{(5.83m)^2}$$

$$|\vec{F}| = \boxed{8.47\times10^{-9}N}$$

a) 1.45×10^{-8} along the $+x$ direction
b) 1.45×10^{-8} along the $-x$ direction
c) 1.45×10^{-8} along the $+y$ direction
d) 1.69×10^{-8} along the $-x$ direction
e) 1.69×10^{-8} along the $-y$ direction



The y components of the forces add up to zero (the same magnitude opposite direction)

While the x components add up to

$$F_{Total} = F_x + F_x = 2|\vec{F}|\cos\theta$$

$$= 2(8.47\times10^{-9}N)\cos(30.96°)$$

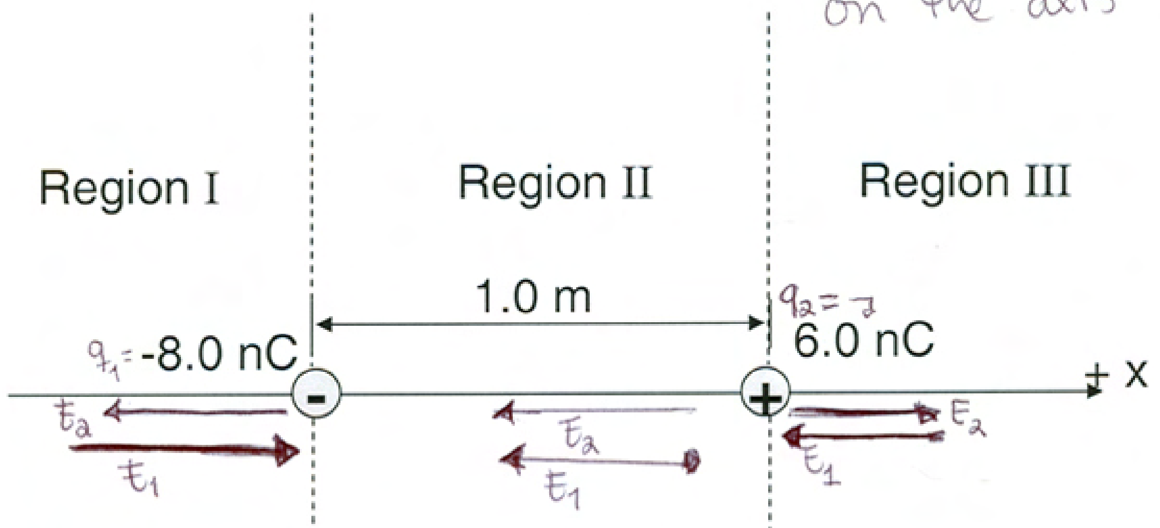
$$= 1.45\times10^{-8}N$$

in the $-x$ direction

$\boxed{b)}$

2.- Two point charges lie on the x-axis. Determine which region(s) along the x axis would contain a point where the electric field is zero other than infinity.

on the axis



$E_1 = \frac{Kq_1}{r^2}$

$E_2 = \frac{Kq_2}{r^2}$

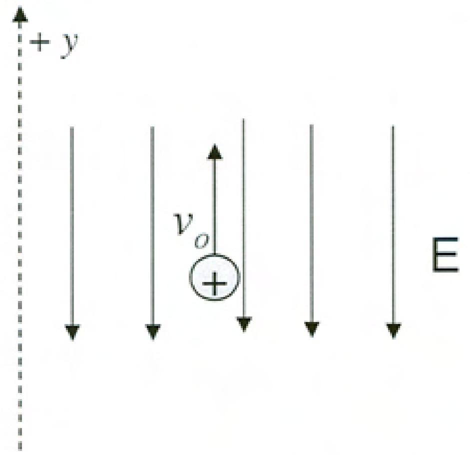
a) Region I
b) Region II
c) Region III
d) Region I and III
e) None

On Region I:
Both electric fields are pointing in opposite direction, so they substract. However, since q_1 is larger in magnitude and on this region we are always closer to it, E_1 is always larger. The never add up to zero

On Region II:
Both field are parallel on the axis. They always add

On Region III:
The electric fields are antiparallel. They do substract. Since q_2 is smaller than q_1, there would be a point on which they add up to zero.

3.- A proton is shot vertically upward with a velocity $v_0 = (2\times10^5\ \text{m/s})\hat{\jmath}$ in a uniform electric field $E = (-500\ \text{N/C})\ \hat{\jmath}$. How far does the proton travel before it is brought momentarily to rest? **(neglect gravity)**



$q_p = 1.602\times10^{-19}\,C$

$m_p = 1.67\times10^{-27}\,kg$

The electric force exerted on the positive charge is downward, and its magnitude is

$$F = qE = (1.602\times10^{-19}C)(500\ N/C)$$

$$F = \underline{8.01\times10^{-17}N}$$

According to Newton's 2nd Law we have

$$F = m_p a \quad \leftarrow \text{solving for } a$$

$$a = \frac{F}{m_p} = \frac{8.01\times10^{-17}N}{1.67\times10^{-27}kg} = \underline{4.79\times10^{10}\ m/s^2}$$

Since it's deceleration, it's negative

a) ∞
b) 200 m
c) 0.25 m
d) 0.84 m
e) 0.42 m

Now, using kinematics we can figure how far the proton will travel

$a = -4.79\times10^{10}\ m/s^2$

$V_o = 2\times10^5\ m/s$

$V_f = 0\ m/s$

$\Delta y = ?$

$$\cancel{V_f^2}^{\,0} = V_o^2 + 2a\Delta y$$

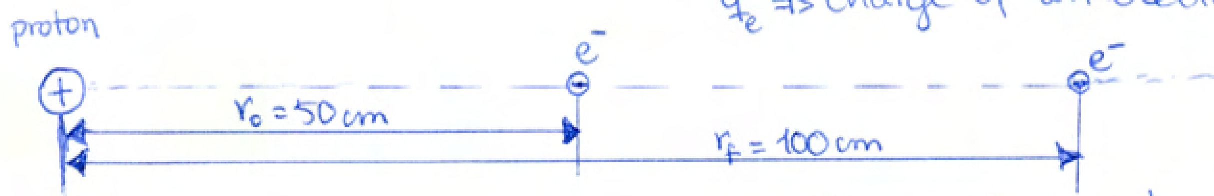
$$-2a\Delta y = V_o^2$$

$$\Delta y = -\frac{V_o^2}{2a} = \frac{(2\times10^5 m/s)^2}{2(-4.79\times10^{10}m/s^2)}$$

$$\boxed{\Delta y = 0.42\ m}$$

4.- How much work, done by an external agent, is required to move an electron from a point 50 cm away from a proton to a point 100 cm away from the same proton?

$q_e \Rightarrow$ charge of an electron



$q_p \Rightarrow$ charge of a proton.

The work done by an external agent is equal to the change in electric potential energy of the electron

$$W_{external} = \Delta U = q_e \Delta V = q_e (V_f - V_o)$$

The potential at its initial position due to the proton is

$$V_o = \frac{K q_p}{r_o}$$

The potential at its final position is:

$$V_f = \frac{K q_p}{r_f}$$

a) $+2.3\times10^{-28}$ J
b) $+6.9\times10^{-28}$ J
c) -2.3×10^{-28} J
d) -6.9×10^{-28} J

e) none of the above

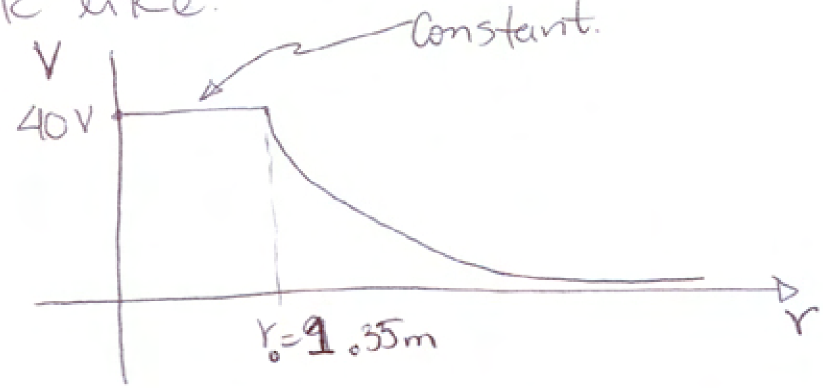
$$W_{external} = q_e \left(\frac{K q_p}{r_f} - \frac{K q_p}{r_o} \right) = q_e K q_p \left(\frac{1}{r_f} - \frac{1}{r_o} \right)$$

$$= (-1.602\times10^{-19} C)\left(9\times10^{9} \frac{N\cdot m^2}{C^2}\right)(+1.602\times10^{-19} C)\left(\frac{1}{1m} - \frac{1}{0.5m}\right)$$

$$W_{external} = +2.3\times10^{-28}\ J$$

5.- An empty hollow metal sphere of radius R= 1.35 m has a potential of 40 V with respect to ground (defined to be zero at infinity) and has a charge of 6.0×10^{-9} C. Find the electric potential at the center of the sphere

The potential for this kind of problem look like.



The potential at the center remains constant once you reach the sphere, since the electric field inside the metal sphere is zero. All the charge is at the outer surface.

a) 0 V

b) 29.6 V

c) 20 V

d) 40 V

e) none of the above

$$\Delta V = \cancel{V_\infty}^{\,0} - V_{center} = -\int \bar{E} \cdot d\vec{l}$$

$$= -\cancel{\int_{\infty}^{r_0} \bar{E}_{inside} \cdot d\vec{l}}^{\,0} - \int_{r_0}^{\infty} \bar{E} \cdot d\vec{l}$$

$$\Delta V = V_{center} = \int_{r_0}^{\infty} \bar{E} \cdot d\bar{l}$$

The potential @ the center is equal to the potential @ r_0

$\boxed{d)}$

6.- An isolated single neutral water molecule (H_2O) has an electric dipole moment of magnitude 6.2×10^{-30} C·m. If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it?

$|\vec{p}| = 6.2\times10^{-30}$ C·m

$|\vec{E}| = 1.5\times10^{4}$ N/C

$\vec{\tau} = \vec{p}\times\vec{E}$

The magnitude becomes by definition

$|\vec{\tau}| = |\vec{p}|\,|\vec{E}|\sin\theta$

The maximum torque that the field can exert is when $\vec{p}$ and $\vec{E}$ are perpendicular, or $\theta = 90°$ ($\sin(90°) = 1$)

a) 1 Nm

b) 4.1×10^{-34} Nm

c) 18.6×10^{-26} Nm

d) 8.2×10^{-34} Nm

e) 9.3×10^{-26} Nm

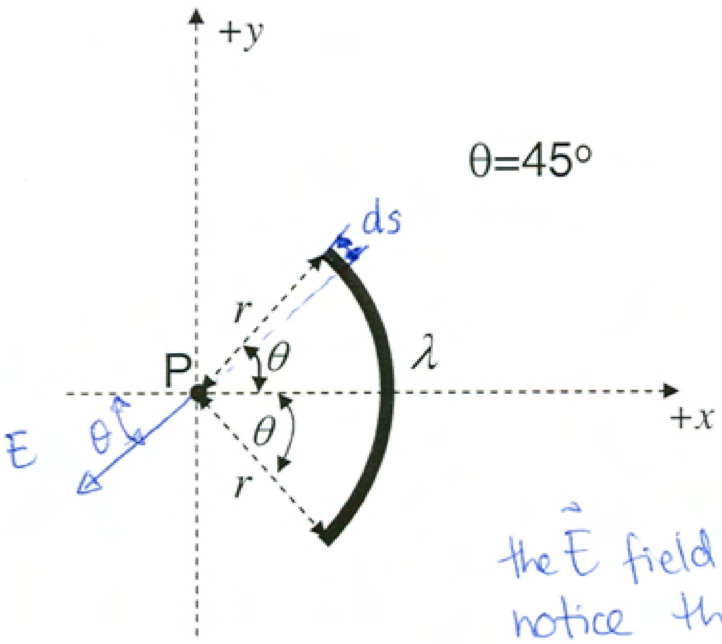
$|\vec{\tau}| = |\vec{p}|\,|\vec{E}|\sin(90°)$

$= |\vec{p}|\,|\vec{E}| = (6.2\times10^{-30}\,\text{C·m})(1.5\times10^{4}\,\text{N/C})$

$|\vec{\tau}| = 9.3\times10^{-26}$ N·m

$\boxed{e)}$

7.- A thin glass rod is bent into a quarter of a circle of radius r. The rod has a uniform linear charge density λ. Find the magnitude of the E field at the point P (at the origin)?



We know that

$$dE = \frac{K\,dq}{r^2}$$

We take a tiny ds along the bent rod which length in polar coordinates is

$$ds = r\,d\theta$$

so that $dq = \lambda ds = \lambda r\,d\theta$

the $\vec{E}$ field is a vector, so we have to notice that the symmetry allows only the x component of the $\vec{E}$ due to dq to survive.

$$dE = \frac{K\,dq}{r^2} = \frac{K\lambda r\,d\theta}{r^2}$$

the x-component is (just) obtained by multiplying this by $\cos\theta$.

$$dE_x = \frac{K\lambda \cancel{r}\,d\theta}{r^{\cancel{2}}}\cos\theta = \frac{K\lambda r\cos\theta\,d\theta}{r^2}$$

a) $E = \dfrac{k\lambda}{r}$

b) $E = \dfrac{k\lambda}{r^2}$

c) $E = \dfrac{2k\lambda}{r^2}$

d) $E = 0$

e) $E = \dfrac{\sqrt{2}k\lambda}{r}$

The r is constant, so we are left with a differential E field that has to be integrated over the whole rod.

$$E_x = \int_{-45°}^{45°} \frac{K\lambda}{r}\cos\theta\,d\theta = \frac{K\lambda}{r}\left[\sin\theta\Big|_{-45°}^{45°}\right]$$

$$= \frac{k\lambda}{r}\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right] = \boxed{\frac{\sqrt{2}k\lambda}{r}} \quad \boxed{e)}$$

8.- Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 6.0 kN $\cdot m^2/C$. Which one of the following answers could describe the charge(s) enclosed in the box?

$$\Phi_E = \frac{Q_{inside}}{\varepsilon_0}$$

$$Q_{inside} = \varepsilon_0 \Phi_E = \left(8.85\times10^{-12}\frac{C^2}{N\cdot m^2}\right)\left(6\times10^{3}\frac{N\cdot m^2}{C}\right)$$

Net charge $\longrightarrow Q_{inside} = +5.31\times10^{-8}\,C$

the only answer that gives a net charge of $+5.31\times10^{-8}$ C is a)

a) $+8.54\times10^{-8}$ C and -3.23×10^{-8} C

$\boxed{a)}$

b) $+6\times10^{-9}$ C

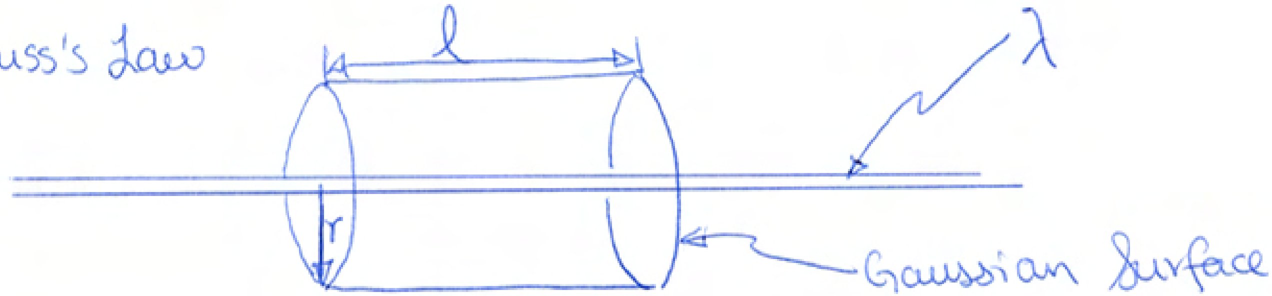
c) -8.54×10^{-8} C and $+3.23\times10^{-8}$ C

d) -6×10^{-9} C

e) none of the above

9.- Consider an infinitely long line of charge of uniform charge density $\lambda = 9$ nC/m. Find the magnitude of the E field at a radial distance of $r = 9$m from the rod.

Using Gauss's Law



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{\varepsilon_0}$$

the closed surface is composed of two end caps and the barrel part of the cylindrical Gaussian surface. Only the barrel contributes to the flux.

$$\cancel{\int_{end\ caps} \vec{E} \cdot dA}^{\,0} + \int_{Barrel} \vec{E} \cdot dA = \frac{Q_{inside}}{\varepsilon_0}$$

a) 9×10^2 N/C

b) 9 N/C

c) 18 N/C

d) 1.01×10^3 N/C

e) 18×10^2 N/C

Since the $\vec{E}$ field points radially the E field is constant over the barrel and perpendicular to it

$$\int_{Barrel} E\, dA = \frac{Q_{inside}}{\varepsilon_0} \qquad Q_{inside} = l\lambda$$

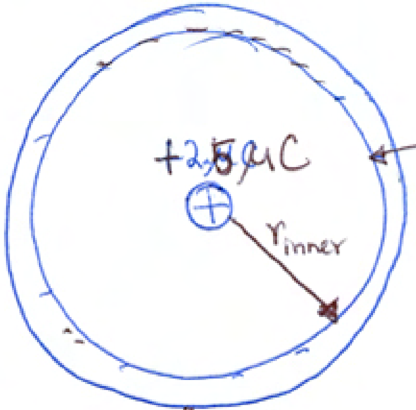
$$E\, A = \frac{Q_{inside}}{\varepsilon_0}$$

$$E\,(2\pi r \cancel{l}) = \frac{\cancel{l}\lambda}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}} = \frac{9\times10^{-9}\,C/m}{2\pi\left(8.85\times10^{-12}\frac{C^2}{N\cdot m^2}\right)(9m)} \stackrel{!}{=} \boxed{17.98\,N/C}$$

$\boxed{c)}$

10.- A positive point charge of magnitude +2.5 μC is at the center of a spherical conducting shell of inner radius 60 cm and outer radius 90 cm. The shell carries a net charge of +3.5 μC. Find the surface charge densities on the inner (σ_{60}) and outer surfaces (σ_{90}) of the shell.



total charge

−2.5μC would be smeared on the inner surface

$$\sigma_{60} = \frac{-2.5\times10^{-6}\,C}{4\pi(r_{inner})^2} = \boxed{-5.53\times10^{-7}\,C/m^2}$$

σ_{90}

On the outer radius, +2.5μC plus +3.5μC would be evenly distributed giving

a) $\sigma_{60} = -2.50\times10^{-6}\ C/m^2$ and $\sigma_{90} = +6.0\times10^{-6}\ C/m^2$
b) $\sigma_{60} = -5.53\times10^{-7}\ C/m^2$ and $\sigma_{90} = +5.89\times10^{-7}\ C/m^2$
c) $\sigma_{60} = -5.53\times10^{-7}\ C/m^2$ and $\sigma_{90} = +7.74\times10^{-7}\ C/m^2$
d) $\sigma_{60} = -5.89\times10^{-7}\ C/m^2$ and $\sigma_{90} = +5.89\times10^{-7}\ C/m^2$
e) $\sigma_{60} = 0$ and $\sigma_{90} = +7.74\times10^{-7}\ C/m^2$

$$\sigma_{90} = \frac{+6.0\,\mu C}{4\pi(r_{outer})^2}$$

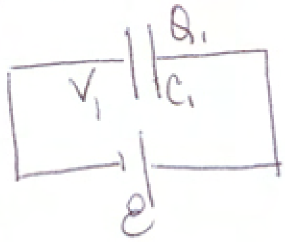
$$\sigma_{90} = \boxed{+5.89\times10^{-7}\,C/m^2}$$

$\boxed{b)}$

11.- An air-filled parallel-plate capacitor has plates of area 40 cm^2, plate separation of 1.0 mm and is charged to a potential difference V_1=100 V. The charging battery is then disconnected, and the plates are pulled apart until the separation is 2.0 mm. What is the new potential difference V_2 between the plates of the capacitor?

First we have

$C_1 \quad V_1 \quad Q_1$



$\boxed{Q_1 = C_1 V_1} \Rightarrow V_1 = \frac{Q_1}{C_1}$

Then we remove the battery and we strart pulling the plates apart:

The charge on the plates remains the same, however the capacitance charges, that means the voltage has to change too. $(Q = CV)$

The new capacitance is $\boxed{C_2 = \frac{\varepsilon_0 A}{d_2} = \frac{\varepsilon_0 A}{2d_1} = \frac{1}{2}\frac{\varepsilon_0 A}{d_1} = \frac{1}{2} C_1}$

half of the old one

a) V_2= 200 V
b) V_2= 50 V
c) V_2= 0V
d) V_2= 100V
e) V_2= 400V

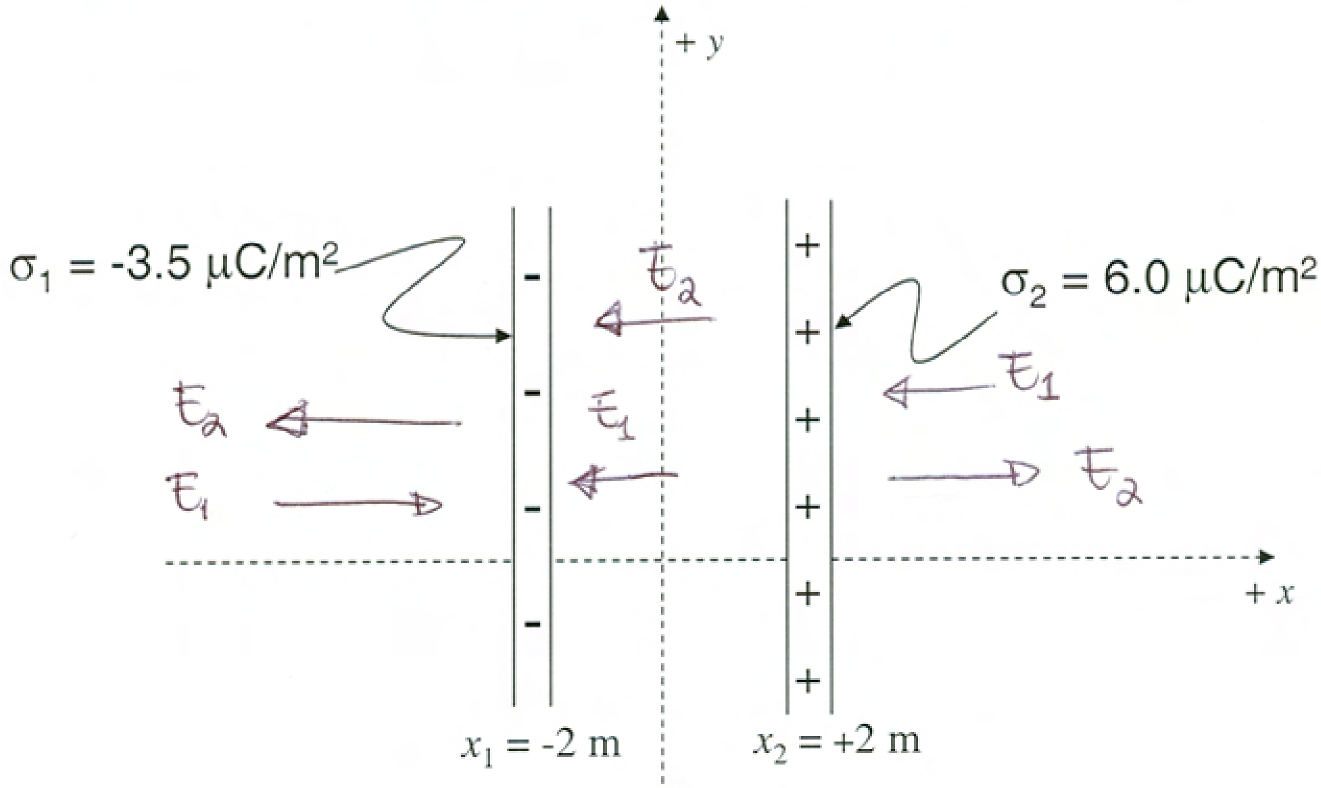
$$Q_1 = C_2 V_2 = \left(\frac{1}{2} C_1\right) V_2$$

$$V_2 = \frac{2Q_1}{C_1} = 2\left(\frac{Q_1}{C_1}\right) = \boxed{2V_1}$$

$$= 2(100V) = \boxed{200\ V.}$$

$\boxed{a)}$

12.- Two infinite (non-conductive) planes lie parallel to each other and to the yz plane. One is at x_1 = -2 m and has a surface charge density of σ_1 = -3.5 μC/m². The other is at x_2 = 2 m and has a surface charge density of σ_2 = 6.0 μC/m². Find the electric field at x = -4 m.



a) 0 MV/m

b) 0.53 MV/m along the $-x$ direction

c) 0.14 MV/m along the $+x$ direction

d) 0.53 MV/m along the $+x$ direction

e) 0.14 MV/m along the $-x$ direction

The electric field is constant on both sides of a non-conducting plane

$$E = \frac{|\sigma|}{2\varepsilon_0}$$

$$E_2 = -\frac{|\sigma_2|}{2\varepsilon_0}\hat{\imath}$$

$$E_1 = +\frac{|\sigma_1|}{2\varepsilon_0}\hat{\imath}$$

adding

$$E_{Total} = \frac{1}{2\varepsilon_0}\left(|\sigma_1| - |\sigma_2|\right)\hat{\imath} = -0.14 \text{ MV/m}\ \hat{\imath}$$

$\boxed{e)}$