Solutions - Codrington

Exam 2 PHYS-241
November 4, 2004

1.- Two 8 1/2” x 11” crib sheets are allowed. It must be of your own creation.

2.- Please print your name on the top edge of the op-scan sheet and sign it.

3.- Use a #2 pencil to fill in your full name, your student identification number,your recitation division number, and finally the answers for problems 1–12.

$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}$$

$$\varepsilon_0 = 8.85\times10^{-12} \ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$$

$$\mu_0 = 4\pi\times10^{-7} \ \frac{\mathrm{N}}{\mathrm{A^2}}$$

$$e = 1.602\times10^{-19}\,\mathrm{C}$$

$c = 2.99792458\times10^8$ m/s (speed of light)

$N_{Avogadro} = 6.022\times10^{23}$ (number of atoms in 12 g of ^{12}C)

$\mathrm{m} \Rightarrow 10^{-3} \quad \mu \Rightarrow 10^{-6} \quad \mathrm{n} \Rightarrow 10^{-9} \quad \mathrm{p} \Rightarrow 10^{-12} \quad \mathrm{f} \Rightarrow 10^{-15}$

$\mathrm{k} \Rightarrow 10^{3} \quad \mathrm{M} \Rightarrow 10^{6} \quad \mathrm{G} \Rightarrow 10^{9} \quad \mathrm{T} \Rightarrow 10^{12} \quad \mathrm{P} \Rightarrow 10^{15}$

For $ax^2 + bx + c = 0$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1. Two copper wires have the same volume, but wire 2 is 10% longer than wire 1 (Hint: If the volume remains constant but the length increases, does the cross-sectional area change?). The ratio of the resistances of the two wires R_2/R_1 is:

Let L_i = length of wire i
A_i = cross-sectional area of wire i
V_i = volume of wire i

A) 1.2
B) 1.1
C) 0.82
D) 0.91
E) 1.0

$$L_2 = (1 + .10)\,L_1$$

$$V_2 = V_1 \Rightarrow \underbrace{L_2}_{(1.1)L_1} A_2 = L_1 A_1 \Rightarrow (1.1) L_1 A_2 = L_1 A_1$$

$$\Rightarrow (1.1) A_2 = A_1 \Rightarrow A_2 = \frac{A_1}{1.1}$$

$$\therefore \frac{L_2}{A_2} = \frac{(1.1) L_1}{\frac{A_1}{1.1}} = (1.1)^2 \frac{L_1}{A_1}$$

$$\therefore \frac{R_2}{R_1} = \frac{\rho_{copper} \frac{L_2}{A_2}}{\rho_{copper} \frac{L_1}{A_1}} = \frac{\rho_{copper} (1.1)^2 \frac{L_1}{A_1}}{\rho_{copper} \frac{L_1}{A_1}} = (1.1)^2 = \boxed{1.21}$$

$$\Rightarrow \boxed{A}$$

2. A charged particle is moving horizontally westward with a velocity of 3.5×10^6 m/s in a region where there is a magnetic field of magnitude 5.6×10^{-5} T directed vertically downward. The particle experiences a force of 7.8×10^{-16} N northward. What is the charge on the particle?

A) $+4.0 \times 10^{-18}$ C
B) -4.0×10^{-18} C
C) $+4.9 \times 10^{-5}$ C
D) -1.2×10^{-14} C
E) $+1.4 \times 10^{-11}$ C

$$\vec{F} = q(\vec{v} \times \vec{B}) \Rightarrow |\vec{F}| = |q|\,\underbrace{|\vec{v} \times \vec{B}|}_{|\vec{v}||\vec{B}|\underbrace{\sin 90^\circ}_{1}} = |q||\vec{v}||\vec{B}|$$

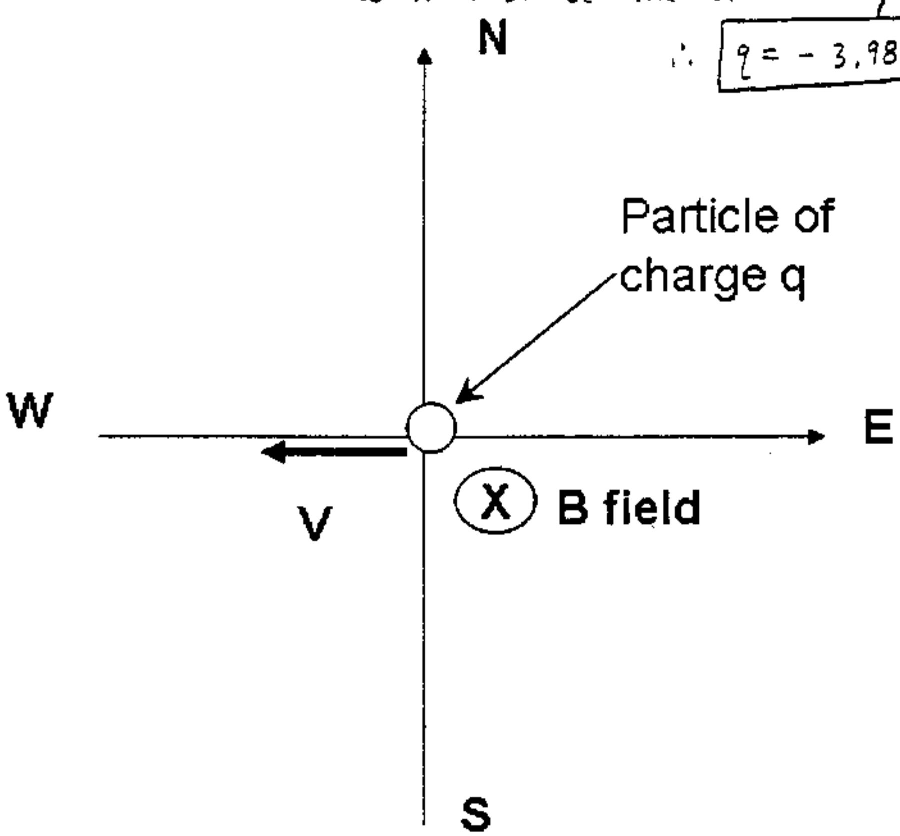
← angle between $\vec{v}, \vec{B}$

$$\Rightarrow |q| = \frac{|\vec{F}|}{|\vec{v}||\vec{B}|} = \frac{7.8 \times 10^{-16}\ \text{N}}{(3.5 \times 10^6\ \frac{\text{m}}{\text{s}})(5.6 \times 10^{-5}\ \text{T})} = 3.98 \times 10^{-18}\ \text{C}$$

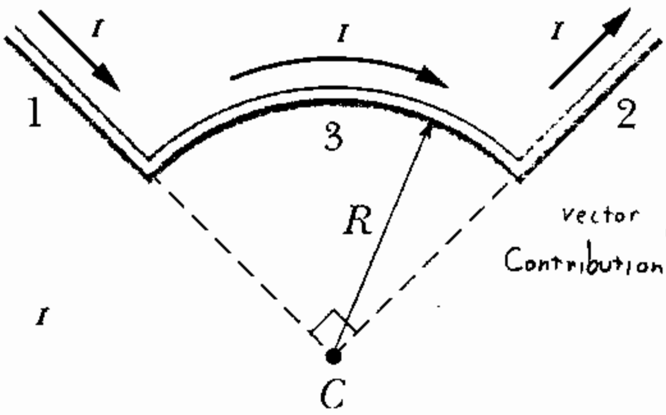
Find sign of q: direction of $\vec{v} \times \vec{B}$ is given by a right-hand rule:
point fingers in direction of 1st vector $\vec{v}$ (wes
curl them in direction of 2nd vector $\vec{B}$ (into pa
thumb points in direction of $\vec{v} \times \vec{B}$ (south)

$\therefore$ $\vec{F} = q(\vec{v} \times \vec{B})$ where $\vec{F}$ points north and $\vec{v} \times \vec{B}$ points south, so it must be the case that $q < 0$.

$\therefore$ $\boxed{q = -3.98 \times 10^{-18}\ \text{C}}$ $\Rightarrow$ $\boxed{B}$



3. The wire in the figure carries a current I and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field $\vec{B}$ does the current produce at C?

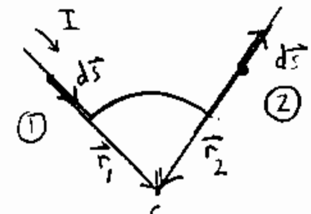


An element of wire $d\vec{s}$ carrying current I makes a contribution

$$d\vec{B} = \frac{\mu_o I}{4\pi} \frac{d\vec{s} \times \vec{r}}{r^3}$$

to the $\vec{B}$-field at C, where $\vec{r}$ is a vector pointing from the element of wire to C.

Contribution from sections ① and ②:



A) $\frac{\mu_o I}{R}\left(\frac{1}{\pi}+\frac{1}{8}\right)$

B) $\frac{90\mu_o I}{4\pi R}$

C) $\frac{\mu_o I}{4\pi R}$

D) $\frac{\mu_o I}{8R}$

E) 0

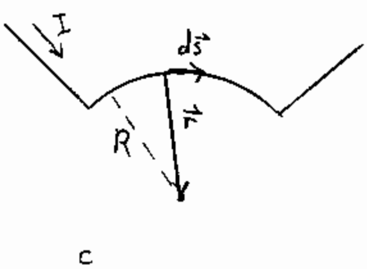
$$|d\vec{B}_1| = \frac{\mu_o I}{4\pi} \frac{|d\vec{s} \times \vec{r}_1|}{(r_1)^3} = \frac{\mu_o I}{4\pi} \frac{|d\vec{s}|\, r_1 \sin 0^\circ}{(r_1)^3} = 0$$

$\boxed{\text{angle between } d\vec{s} \text{ and } \vec{r}_1 \text{ is } 0^\circ \text{ over section ①}}$

$\boxed{\text{angle between } d\vec{s} \text{ and } \vec{r}_2 \text{ is } 180^\circ \text{ over section ②}}$

$$|d\vec{B}_2| = \frac{\mu_o I}{4\pi} \frac{|d\vec{s} \times \vec{r}_2|}{(r_2)^3} = \frac{\mu_o I}{4\pi} \frac{|d\vec{s}|\, r_2 \sin 180^\circ}{(r_2)^3} = 0$$

Contribution from section ③:



$$\overbrace{|d\vec{B}_3|}^{dB_3} = \frac{\mu_o I}{4\pi} \frac{|d\vec{s} \times \vec{r}|}{r^3} = \frac{\mu_o I}{4\pi} \frac{\overbrace{|d\vec{s}|}^{ds}\,\overbrace{|\vec{r}|}^{r}\,\overbrace{\sin 90^\circ}^{1}}{r^3}$$

$$= \frac{\mu_o I\, r\, ds}{4\pi\, r^3} = \frac{\mu_o I\, ds}{4\pi\, r^2}$$

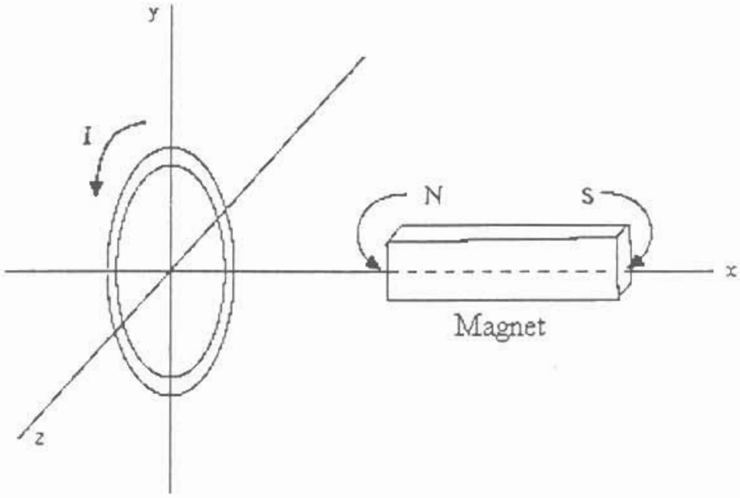
$$B_3 = \int_{arc} dB_3 = \int_{arc} \frac{\mu_o I\, ds}{4\pi r^2} = \int_{arc} \frac{\mu_o I\, ds}{4\pi R^2}$$

$$= \frac{\mu_o I}{4\pi R^2} \boxed{\int_{arc} ds} = \frac{\mu_o I\, R\phi}{4\pi R^2} = \frac{\mu_o I \phi}{4\pi R}$$

$\int_{arc} ds$ = length of arc $= R\phi$ ← ϕ must be in radians

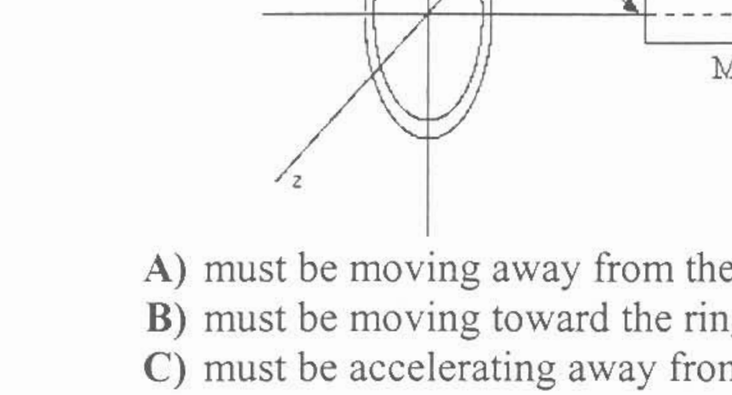
$$\therefore\ |\vec{B}| = |\vec{B}_3| = \frac{\mu_o I \phi}{4\pi R} \underset{\phi = \frac{\pi}{2}}{=} \frac{\mu_o I \left(\frac{\pi}{2}\right)}{4\pi R} = \boxed{\frac{\mu_o I}{8R}} \Rightarrow \boxed{D}$$

4. A copper ring lies in the *yz* plane as shown. The magnet's long axis lies along the *x* axis. Induced current flows through the ring as indicated. The magnet



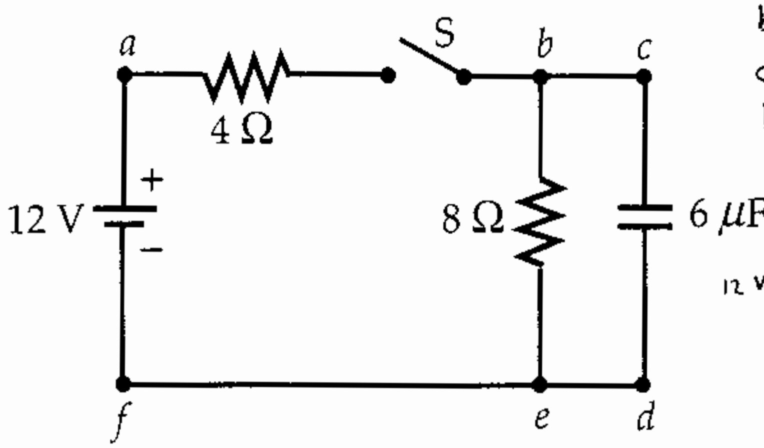
A) must be moving away from the ring.
B) must be moving toward the ring.
C) must be accelerating away from the ring
D) is not necessarily moving.
E) must remain stationary to keep the current flowing.

Consider the case where the magnet is moving toward the ring with velocity $\vec{v}$. The lines of the $\vec{B}$-field come out of the North Pole of the magnet and go into the south pole, so $\vec{B}$ points to the left at the position of the ring.

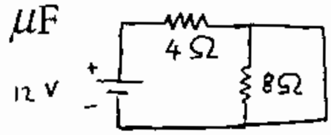


Since the magnet is moving toward the ring, the $\vec{B}$-field is getting stronger at the position of the right and thus the magnetic flux $\Phi_B = \iint_{ring} \vec{B} \cdot d\vec{A}$ is getting stonger as well. Lenz's Law states that the induced current flows in a direction as to oppose the change in the magnetic flux. The magnetic flux is getting stronger; to oppose this change, we want to weaken the magnetic flux. To weaken the magnetic flux, we want to weaken the overall $\vec{B}$-field, which can be done by making the induced $\vec{B}$-field, $\vec{B}_{ind}$, point in the opposite direction to the original $\vec{B}$-field (i.e. to the right). By a right-hand rule, if we point our thumb in the direction of the induced $\vec{B}$-field, our fingers curl in the direction of the induced current. The direction we obtain for the induced current agrees with that indicated above, hence the magnet is moving $\boxed{\text{toward the ring.}}$ $\Rightarrow$ $\boxed{B}$

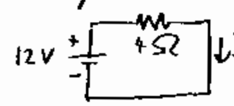
5. The 6-μF capacitor in the circuit shown in the figure is initially uncharged. Find the current through the 4-Ω resistor and the current through the 8-Ω resistor:
(i) immediately after the switch is closed,
(ii) a long time after the switch is closed,
(iii) Find the charge on the capacitor a long time after the switch is closed.



(i) Immediately after switch is closed, there is no charge on capacitor, so by $q = CV$, the voltage across the capacitor is zero, i.e. the capacitor behaves like a short circuit (i.e a piece of wire)



⇒ no current flows thru 8 Ω resistor so may as well remove it:

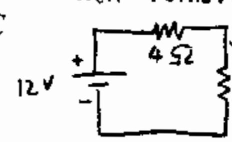


↓I $12\text{ V} - I(4\Omega) = 0$

$\Rightarrow I_{4\Omega} = I = \frac{12\text{ V}}{4\Omega} = \boxed{3A}$

$I_{8\Omega} = \boxed{0}$

(ii) A long time after the switch is closed, the capacitor is fully charged; no more charge flows onto the plates of the capacitor, so the capacitor behaves like an open circuit, and we may as well remove it:



↓I $12\text{ V} - I(4\Omega) - I(8\Omega) =$

$\Rightarrow I = \frac{12\text{ V}}{12\Omega} = 1A$

$I_{4\Omega} = I_{8\Omega} = I = \boxed{1A}$

A) (i) $I_{4\Omega} = I_{8\Omega} = 1A$; (ii) $I_{4\Omega} = 3A$ and $I_{8\Omega} = 0A$; (iii) $0\mu C$

B) (i) $I_{4\Omega} = I_{8\Omega} = 1A$; (ii) $I_{4\Omega} = I_{8\Omega} = 1A$; (iii) $48\mu C$

C) (i) $I_{4\Omega} = 3A, I_{8\Omega} = 0A$; (ii) $I_{4\Omega} = 3A$ and $I_{8\Omega} = 0A$; (iii) $0\mu C$

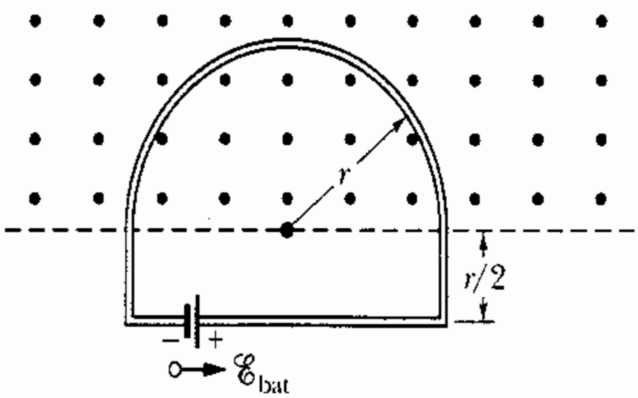
D) (i) $I_{4\Omega} = 3A, I_{8\Omega} = 0A$; (ii) $I_{4\Omega} = I_{8\Omega} = 1A$; (iii) $48\mu C$

E) (i) $I_{4\Omega} = 0A, I_{8\Omega} = 3A$; (ii) $I_{4\Omega} = I_{8\Omega} = 3A$; (iii) $144\mu C$

(iii) Voltage across capacitor = Voltage across 8Ω resistor = $I_{8\Omega}(8\Omega) = (1A)(8\Omega) = 8\text{ V}$

Charge on capacitor = C (Voltage across capacitor) = $(6\,\mu F)(8V) = \boxed{48\,\mu C} \Rightarrow \boxed{D}$

6. The figure shows a conducting loop consisting of a half-circle of radius r = 0.20 m and three straight sections. The half-circle lies in a uniform magnetic field of $\vec{B}$ that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf $\varepsilon_{bat} = 2.0$ V is connected to the loop. The resistance of the loop is 2.0 Ω.
(i) What is the magnitude of the emf ε_{ind} induced around the loop by field $\vec{B}$ at $t = 10$ s?
(ii) What are the magnitude and direction of the current in the loop at $t = 10$ s?



A) (i) 1.3 V; (ii) 0.63 A clockwise

B) (i) 1.3 V; (ii) 0.63 A counterclockwise

C) (i) 0 V; (ii) 0 A

D) (i) 5.2 V; (ii) 1.6 A clockwise

E) (i) 5.2 V; (ii) 1.6 A counterclockwise

The magnetic flux thru the loop is:

$$\Phi_B = \int_{loop} \vec{B}\cdot d\vec{A} = B\frac{\pi r^2}{2}$$

$\boxed{\text{only that part of the loop where } B \neq 0 \text{ contributes to the mag. flux}}$

The emf induced in the loop at t = 10 s is

$$\varepsilon_{ind} = -\frac{d}{dt}(N\Phi_B) = -\frac{d}{dt}\left(B\frac{\pi r^2}{2}\right)$$

N = # turns = 1

$$= -\left.\frac{dB}{dt}\right|_{t=10\,s}\frac{\pi r^2}{2} = -\left.\frac{d}{dt}\left(4t^2+2t+3\right)\right|_{t=10\,s}\frac{\pi r^2}{2}$$

$$= -\left.(8t+2)\right|_{t=10\,s}\frac{\pi r^2}{2} = -(8(10)+2)\frac{\pi(.2\,m)^2}{2}$$

$$= -5.15\text{ V} \Rightarrow |\varepsilon_{ind}| = \boxed{5.15\text{ V}}$$

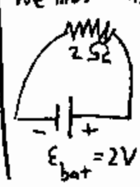
Magnitude of induced current is

$$i_{ind} = \frac{\varepsilon_{ind}}{\left(\text{total resistance of loop}\right)} = \frac{5.15\text{ V}}{2\,\Omega} = 2.576\text{ A}$$

Find direction of induced current: $\Phi_B = B\frac{\pi r^2}{2} = (4t^2+2t+3)\frac{\pi r^2}{2}$, so $|\Phi_B|\uparrow$ with time.

By Lenz's Law, the induced current flows in a direction so as to oppose the change in $|\Phi_B|$. Since $|\Phi_B|\uparrow$ with time, we can oppose this change by weakening the overall $\vec{B}$ field, which can be done by making the induced $\vec{B}$-field, $\vec{B}_{ind}$, point in the opposite direction to $\vec{B}$ (i.e. into the page). By a right hand rule, if we point our thumb in the direction of the induced $\vec{B}$-field (i.e. into the page), our fingers curl in the direction of the induced current, which will be clockwise in this case. ∴ $i_{ind} = 2.576$ A, CW.

We must also consider the current due to the battery:



$$i_{bat} = \frac{\varepsilon_{bat}}{R} = \frac{2\text{ V}}{2\,\Omega} = 1\text{ A, CCW}$$

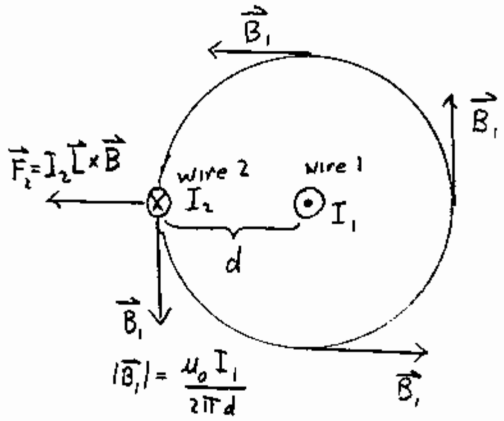
$i_{total} = i_{ind} + i_{bat} = 2.576$ A, CW + 1 A, CCW

$= 2.576$ A, CW − 1 A, CW = $\boxed{1.576\text{ A, CW}}$ ⇒ $\boxed{D}$

7. Two long, straight, parallel wires 11 cm apart carry currents of equal magnitude I. They repel each other with a force per unit length of 4.2 nN/m. Are the currents "parallel" or "antiparallel"? What is the magnitude of the current I?

Consider the case where the currents are antiparallel.

A) antiparallel; I=0.096A
B) parallel; I=0.0023A
C) antiparallel; I=0.0023A
D) parallel; I=0.048A
E) antiparallel; I=0.048A



By a right-hand rule, if we point our thumb in the direction of the current in wire 1, I_1, our fingers curl in the direction of the $\vec{B}$-field due to wire 1, which at the position of wire 2, points down. Wire 2 is thus carrying a current in the presence of a $\vec{B}$-field, so it experiences a force $\vec{F}_2 = I_2 \vec{L} \times \vec{B}_1$, where the vector $\vec{L}$ represents a length $|\vec{L}|$ of wire 2 (the direction of $\vec{L}$ is taken to be in the same direction as the current in wire 2, i.e. into the page). Since the angle between $\vec{L}$ and $\vec{B}_1$ is 90°

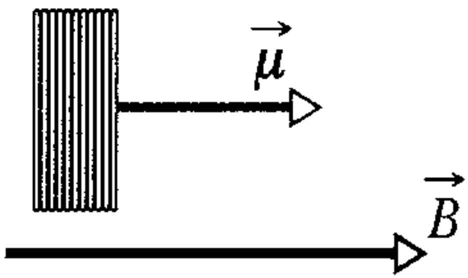
$$|\vec{F}_2| = I_2 |\vec{L} \times \vec{B}_1| = I_2 |\vec{L}| \underbrace{|\vec{B}_1|}_{= \frac{\mu_0 I_1}{2\pi d}} \underbrace{\sin 90°}_{=1} = \frac{\mu_0 I_1 I_2 |\vec{L}|}{2\pi d}$$

The direction of the force on wire 2, $\vec{F}_2 = I_2 \vec{L} \times \vec{B}$, is found by a right-hand rule:
Point your fingers in the direction of the 1st vector $\vec{L}$ (into page)
Curl them in the direction of the 2nd vector $\vec{B}$ (down), then your thumb points in the direction of $\vec{L} \times \vec{B}$ (to the left), which is also the direction of $\vec{F}_2$.
Thus the force on wire 2 points to the left, and it follows that the force between the two wires is repulsive. Thus our initial guess that the currents are $\boxed{\text{antiparallel}}$ was correct.

The force per unit length is $\frac{|\vec{F}_2|}{|\vec{L}|} = \frac{\mu_0 I_1 I_2}{2\pi d} \underset{I_1 = I_2 = I}{=} \frac{\mu_0 I^2}{2\pi d}$

$$\Rightarrow \quad I = \left(\frac{2\pi d}{\mu_0} \frac{|\vec{F}_2|}{|\vec{L}|}\right)^{1/2} = \left(\frac{2\pi\ (.11\ m)}{(4\pi \times 10^{-7}\ \frac{T\cdot m}{A})} (4.2 \times 10^{-9}\ \tfrac{N}{m})\right)^{1/2} = \boxed{.048\ A} \quad \Rightarrow \boxed{E}$$

8. The figure shows a circular coil with 250 turns, an area A of 2.52 x 10^{-4} m^2, and a current of 100 μA. The coil is at rest in a uniform magnetic field of magnitude $B = 0.85$ T, with its magnetic dipole moment $\vec{\mu}$ initially aligned with $\vec{B}$. How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its initial orientation, so that $\vec{\mu}$ is perpendicular to $\vec{B}$ and the coil is again at rest?



A) -10.72 μJ
B) -5.36 μJ
C) 0 μJ
D) 10.72 μJ
E) 5.36 μJ

Potential energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is

$$U(\theta) = -\vec{\mu}\cdot\vec{B} = -\underbrace{|\vec{\mu}|}_{= NIA}|\vec{B}|\cos\theta$$

θ: angle between $\vec{\mu}, \vec{B}$

Let τ_{ext} be the torque applied by an external agent. Then

$$\tau_{ext} = \Delta \text{Energy} = U_{final} - U_{initial} = \underbrace{U(90°)}_{= -NIA|\vec{B}|\underbrace{\cos 90°}_{0}} - \underbrace{U(0°)}_{= -NIA|\vec{B}|\cos 0°}$$

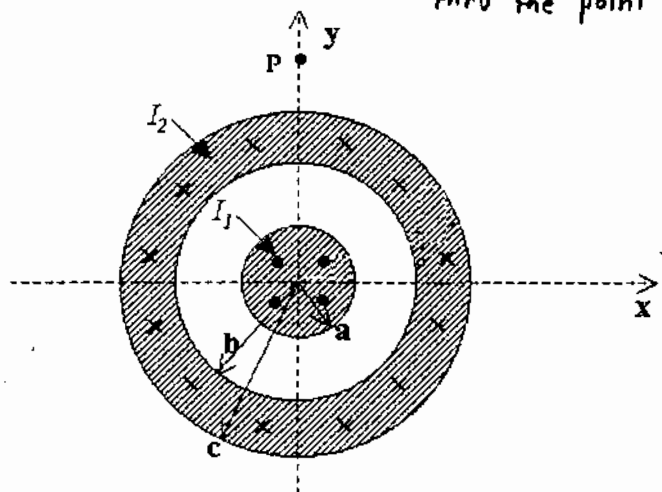
$$= NIA|\vec{B}|$$

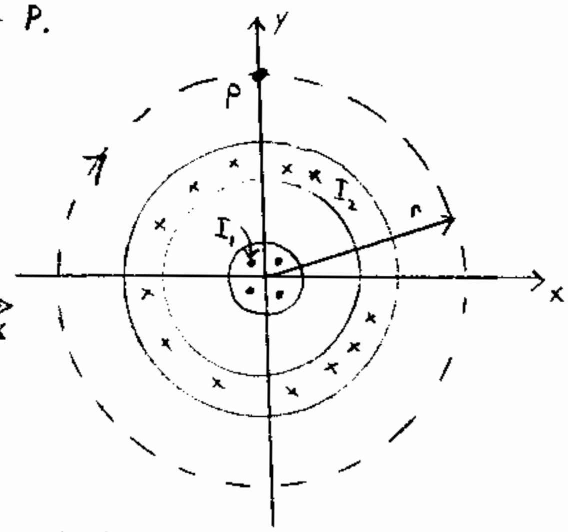
$$= (250 \text{ turns})(100\times10^{-6} A)(2.52\times10^{-4} m^2)(.85 T)$$

$$= \boxed{5.355\times10^{-6} \ N\cdot m} \Rightarrow \boxed{E}$$

9. Two very long coaxial cylindrical conductors are shown in cross-section below. The inner cylinder has radius a = 2 cm and carries a total current of I_1 = 1.2 A in the positive z-direction (pointing out of the page). The outer cylinder has an inner radius b = 4 cm, outer radius c = 6 cm and carries a current of I_2 = 2.4 A in the negative z-direction (pointing into the page). You may assume that the current is uniformly distributed over the cross-sectional area of the conductors. What are the magnitude and direction of the magnetic field B at point P which lies on the y axis at y = 8 cm?

We can find the $\vec{B}$-field at P by defining an Amperian loop of radius r = 8 cm passing thru the point P.





Then by Ampere's Law, $\oint_{loop} \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

A) $0T$

B) $9\times10^{-6}T$ in the negative x direction

C) $9\times10^{-6}T$ in the positive x direction

D) $3\times10^{-6}T$ in the negative x direction

E) $3\times10^{-6}T$ in the positive x direction

where $d\vec{s}$ is an element of arclength along the loop. Since the $\vec{B}$-field due to this current distribution only has a component tangential to the loop, $\vec{B}\cdot d\vec{s} = B\,ds\cos\theta = B\,ds$, where B is the component of the B-field in the direction of integration, which we take to be clockwise (so at P, B is the component of the $\vec{B}$-field in the positive x direction. Thus

$$\mu_0 I_{encl} = \int_{loop} \underbrace{\vec{B}\cdot d\vec{s}}_{=B\,ds} = B \boxed{\int_{loop} ds}_{=\text{distance along loop} = 2\pi r} = B\cdot 2\pi r \Rightarrow B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0 (I_2 - I_1)}{2\pi r}$$

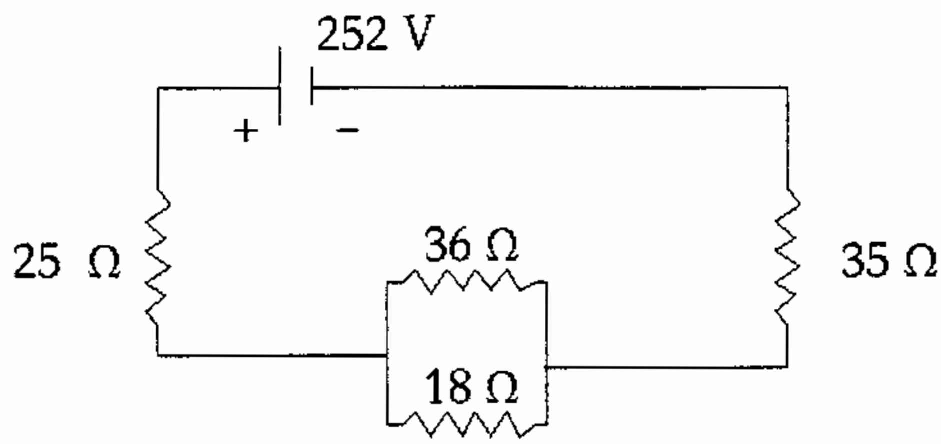
The sign of the contribution of a current to I_{encl} is determined by a right-hand rule: curl fingers in direction of integration (in this case clockwise), then thumb points in the direction of the positive sense of current (in this case, into the page). So $I_{encl} = I_2 - I_1$

$$B = \frac{\mu_0 (I_2 - I_1)}{2\pi r} = \frac{(4\pi\times10^{-7}\ \frac{T\cdot m}{A})(2.4 - 1.2\ A)}{2\pi\,(.08\ m)}$$

$= \boxed{3.0\times10^{-6}\ T \text{ in the positive x dir.}}$

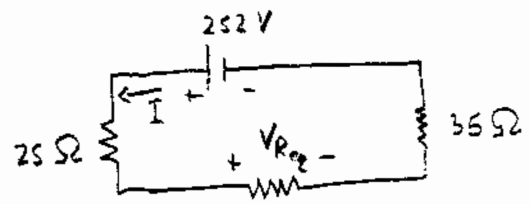
$\Rightarrow$ $\boxed{E}$

10. In the circuit shown, the power dissipated in the 18-Ω resistor is



A) 0.15 kW
B) 98 W
C) 33 W
D) 0.33 kW
E) 47 W

Combining the 36 Ω and 18 Ω resistors (these are in parallel)



$$R_{eq} = \frac{1}{\frac{1}{36}+\frac{1}{18}} = \frac{1}{\frac{1+2}{36}} = \frac{36}{3} = 12\ \Omega$$

Find current I using Kirchoff's Voltage Law:

$$252 - I(25\ \Omega) - I(12\ \Omega) - I(35\ \Omega) = 0$$

$$\Rightarrow 252\ V = I\,(72\ \Omega) \Rightarrow I = \frac{252\ V}{72\ \Omega} = 3.5\ A$$

Voltage across R_{eq}:

$$V_{R_{eq}} = I(R_{eq}) = (3.5\ A)(12\ \Omega) = 42\ V$$

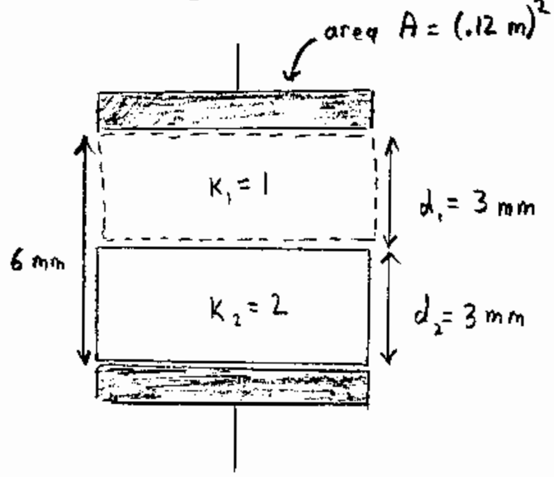
Voltage across 18 Ω resistor same as $V_{R_{eq}}$, since resistors in parallel have the same voltage across each, which is the same as the voltage across the equivalent resistance.

$$\therefore V_{18\Omega} = V_{R_{eq}} = 42\ V$$

Power dissipated in 18 Ω resistor is

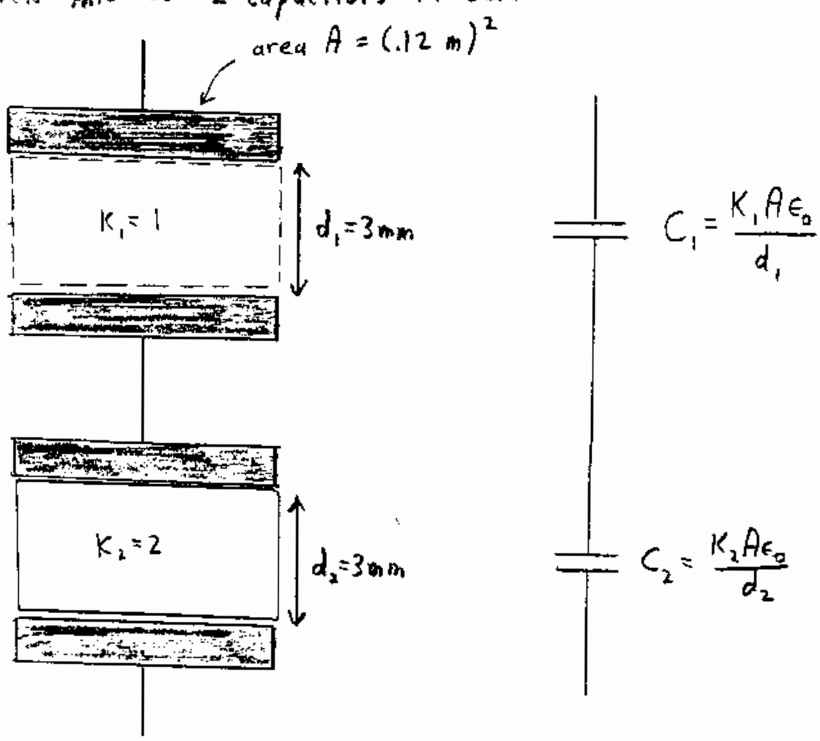
$$P_{18\Omega} = \frac{(V_{18\Omega})^2}{18\ \Omega} = \frac{(42\ V)^2}{18\ \Omega} = \boxed{98\ W} \Rightarrow \boxed{B}$$

11. A parallel-plate capacitor has square plates of side 12 cm and a separation of 6.0 mm. A dielectric slab of constant $\kappa = 2.0$ has the same area as the plates but has a thickness of 3.0 mm. What is the capacitance of this capacitor with the dielectric slab between its plates?



A) 28 pF
B) 21 pF
C) 16 pF
D) 37 pF
E) 53 pF

We can view this as 2 capacitors in series:



Combine capacitors in series

$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{1}{\frac{1}{\frac{K_1 A\epsilon_0}{d_1}}+\frac{1}{\frac{K_2 A\epsilon_0}{d_2}}} = \frac{A\epsilon_0}{\frac{d_1}{K_1}+\frac{d_2}{K_2}} = \frac{(.12\ m)^2(8.85\times10^{-12}\ \frac{C^2}{Nm^2})}{\frac{3.0\times10^{-3}\ m}{1}+\frac{3.0\times10^{-3}\ m}{2.0}}$$

$$= 28.32\times10^{-12}\ F \cong \boxed{28\ pF} \Rightarrow \boxed{A}$$

12. An electric field of 3.0 kV/m is perpendicular to a magnetic field of 0.20 T. An electron moving in a direction perpendicular to both $\vec{E}$ and $\vec{B}$ is not deflected if it has a velocity of

A) 6 km/s
B) 9 km/s
C) 12 km/s
D) 15 km/s
E) 6.7 m/s

Particle not deflected $\Rightarrow$ force $\vec{F} = 0$

$$\therefore \quad 0 = \vec{F} = q\vec{E} + q\,\vec{v} \times \vec{B} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\Rightarrow \quad \vec{E} + \vec{v} \times \vec{B} = 0 \quad \Rightarrow \quad \vec{E} = -\vec{v} \times \vec{B}$$

angle between $\vec{v}$ and $\vec{B}$ is $90°$

$$\Rightarrow \quad |\vec{E}| = |-\vec{v} \times \vec{B}| = |\vec{v} \times \vec{B}| = |\vec{v}||\vec{B}| \underbrace{\sin 90°}_{=1}$$

$$\Rightarrow \quad |\vec{v}| = \frac{|\vec{E}|}{|\vec{B}|} = \frac{(3.0 \times 10^3 \ \frac{V}{m})}{.20 \ T} = 15 \times 10^3 \ \frac{m}{s} = \boxed{15 \ \frac{km}{s}} \quad \Rightarrow \quad \boxed{D}$$

Answer Key

1. A
2. B
3. D
4. B
5. D
6. D
7. E
8. E
9. E
10. B
11. A
12. D