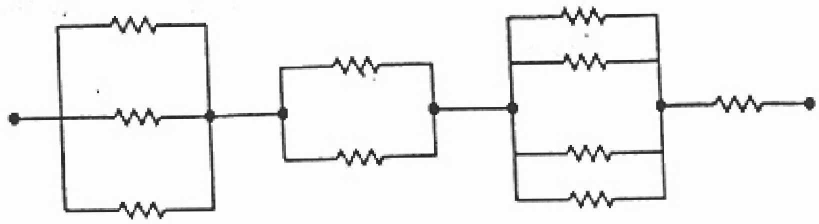
Physics 241
Exam II
March 27, 2002
Solutions - Codrington

1. (10 points) Each of the resistors in the diagram is 12 Ω. The resistance of the entire circuit is:

(A)( ) 5.76 Ω
$\boxed{\text{(B)( ) 25 Ω}}$
(C)( ) 48 Ω
(D)( ) 120 Ω
(E)( ) None of these



$$\frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} = \frac{R}{3} \qquad \frac{1}{\frac{1}{R}+\frac{1}{R}} = \frac{R}{2} \qquad \frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} = \frac{R}{4} \qquad R$$

$$R\left(\tfrac{1}{3}+\tfrac{1}{2}+\tfrac{1}{4}+1\right) = R\left(\frac{4+6+3+12}{12}\right) = \frac{25R}{12} = \frac{25(12\,\Omega)}{12} = 25\,\Omega$$

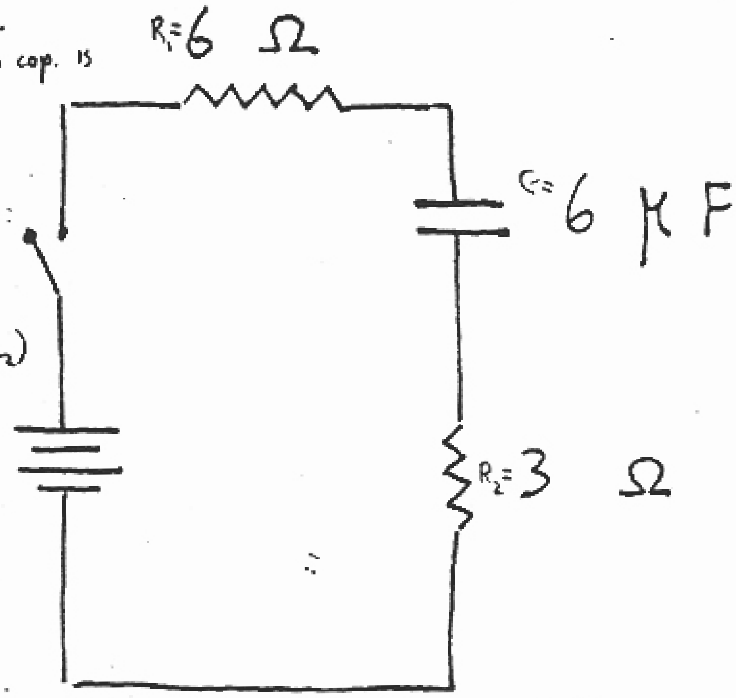
2. (10 points) In the circuit shown, the capacitor is initially uncharged. V = 9 Volts. At time $t = 0$, switch S is closed. If τ denotes the time constant, the approximate current through the 3 Ω resistor when $t = \tau/100$ is:

Final voltage across capacitor is same as case where resistors are not present (resistors only slow rate at which battery can load charge onto capacitor.

(A)( ) 3/8 A
(B)( ) 1/2 A
(C)( ) 3/4 A
$\boxed{\text{(D)( ) 1 A}}$
(E)( ) 3/2 A

Final charge on cap. is

$q_f = CV$



S

V

$$q(t) = q_f\left(1 - e^{-t/\tau}\right)$$

$$i(t) = \frac{dq}{dt} = \frac{q_f}{\tau} e^{-t/\tau} \quad \text{where } \tau = C(R_1+R_2)$$

$$= \frac{(CV)}{C(R_1+R_2)} e^{-(\tau/100)/\tau}$$

$$= \frac{V}{R_1+R_2} e^{-\frac{1}{100}}$$

$$= \frac{9\,V}{6+3\,\Omega} e^{-\frac{1}{100}} = .99\,A \doteq 1\,A$$

3. (10 points) At one instant an electron (charge $= -1.6 \times 10^{-19}$ C) is moving in the xy plane, the components of its velocity being $v_x = 5 \times 10^5$ m/s and $v_y = 3 \times 10^5$ m/s. A magnetic field of 0.8 T is in the positive z direction. At that instant the magnitude of the magnetic force on the electron is:

(A)( ) 0
(B)( ) 3.8×10^{-14} N
(C)( ) 5.1×10^{-14} N
(D)( ) 6.4×10^{-14} N
$\boxed{\text{(E)( ) } 7.5 \times 10^{-14}\text{ N}}$

$$\vec{B} = .8\hat{k}\,T \qquad \vec{v} = 5\times10^5\,\hat{\imath} + 3\times10^5\,\hat{\jmath}\ \text{m/s}$$

$$\vec{v}\times\vec{B} = (5\hat{\imath}+3\hat{\jmath})\times(.8\hat{k}) \times 10^5$$

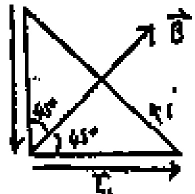
$$= 4\,\underbrace{\hat{\imath}\times\hat{k}}_{-\hat{\jmath}} + 2.4\,\underbrace{\hat{\jmath}\times\hat{k}}_{\hat{\imath}} \times 10^5\ \frac{T\cdot m}{s}$$

$$|\vec{F}| = |q||\vec{v}\times\vec{B}| = (1.6\times10^{-19})\left(\sqrt{(2.4)^2+(-4)^2}\times10^5\right)$$

$$= 7.46\times10^{-14}\ N$$

4. (10 points) A loop of wire carrying a current of 2.0 A is in the shape of a right triangle with two equal sides, each 15 cm long. A 0.7 T uniform magnetic field is in the plane of the triangle and is perpendicular to the hypotenuse. The resultant magnetic force on the two sides has a magnitude of:

(A)( ) 0
(B)( ) 0.21 N
$\boxed{\text{(C)( ) 0.30 N}}$
(D)( ) 0.41 N
(E)( ) 0.51 N



$\vec{F} = i\vec{L}_1 \times \vec{B} + i\vec{L}_2 \times \vec{B} = iL_1B \underbrace{\sin 45°}_{\frac{1}{\sqrt{2}}}$ (out of page) $+ iL_2B\underbrace{\sin 135°}_{\frac{1}{\sqrt{2}}}$ (out of page)

$|\vec{F}| = \frac{2}{\sqrt{2}} iLB = \sqrt{2}(2.A)(.15\ m)(.7\ T) = .297$

5. (10 points) Two parallel wires, 4 cm apart, carry currents of 2 A and 4 A respectively, in opposite directions. The force per unit length in N/m of one wire on the other is:

Let $\vec{L}$ be a vector pointing out of the page representing a length of wire 2

$i_1 \otimes \xleftrightarrow{d} \odot i_2$ $\quad \vec{F} = i_2\vec{L} \times \vec{B}_1 \quad |\vec{F}| = i_2 L B_1 = i_2 L\left(\frac{\mu_0 i_1}{2\pi d}\right)$ (repulsive)

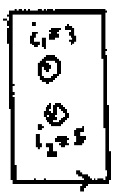
$B_1 = \frac{\mu_0 i_1}{2\pi d}$

(A)( ) 1×10^{-3}, repulsive
(B)( ) 1×10^{-3}, attractive
$\boxed{\text{(C)( ) } 4 \times 10^{-5}\text{, repulsive}}$
(D)( ) 4×10^{-5}, attractive
(E)( ) none of these

$\frac{|\vec{F}|}{L} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{(4\pi \times 10^{-7}\ \frac{T\cdot m}{A})(2\ A)(4\ A)}{2\pi(.04\ m)} = 4.0 \times 10^{-5}$ N repulsive.

6. (10 points) Two long straight wires enter a room through a window. One carries a current of 3.0 A into the room while the other carries a current of 5.0 A out. The magnitude in T·m of the path integral $\oint \vec{B} \cdot d\vec{s}$ around the window frame is:

$\boxed{\text{(A)( ) } 2.5 \times 10^{-6}}$
(B)( ) 3.8×10^{-6}
(C)( ) 6.3×10^{-6}
(D)( ) 1.0×10^{-5}
(E)( ) none of these



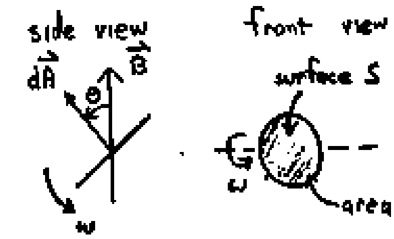
$\oint \vec{B} \cdot d\vec{s} = \mu_0(i_1 - i_2)$

$|\oint \vec{B} \cdot d\vec{s}| = |\mu_0(i_1 - i_2)| = \mu_0|i_1 - i_2|$

$= (4\pi \times 10^{-7}\ \frac{T\cdot m}{A})|3 - 5\ A| = 2.51 \times 10^{-6}\ \frac{T\cdot m}{A}$

7. (10 points) A single loop of wire with a radius of 7.5 cm rotates about a diameter in a uniform magnetic field of 1.6 T. The axis of rotation is perpendicular to the magnetic field. To produce a maximum emf of 1.0 V, it should rotate at:

(A)( ) 0
(B)( ) 2.7 rad/s
(C)( ) 5.6 rad/s
$\boxed{\text{(D)( ) 35 rad/s}}$
(E)( ) 71 rad/s



Assume $\theta = 0$ at $t = 0 \Rightarrow \theta = \omega t$

Flux thru area A

$\Phi_B = \int_S \vec{B} \cdot d\vec{A}$

$= \int_S |\vec{B}|\, dA \cos \theta$ ← ωt

$= \int_S |\vec{B}|\, dA \cos \omega t$

$= |\vec{B}| \cos \omega t \underbrace{\boxed{\int_S dA}}_{A}$

$= |\vec{B}| A \cos \omega t$

$\varepsilon = -\frac{d}{dt}(\underset{N=1}{N}\Phi_B) = -\frac{d}{dt}(|\vec{B}| A \cos \omega t) = -|\vec{B}| A(-\omega \sin \omega t) = \underbrace{|\vec{B}| A \omega}_{\max |\varepsilon|} \sin \omega t$

$\Rightarrow \max |\varepsilon| = |\vec{B}| \overset{\pi r^2}{A}\, \omega = |\vec{B}|(\pi r^2)\omega$

$\Rightarrow \omega = \frac{\max |\varepsilon|}{|\vec{B}| \pi r^2} = \frac{1.0\ V}{(1.6\ T)\pi(.075\ m)^2} = 35.36\ \frac{rad}{s}$

8. (10 points) A 6.0 mH inductor and a 3.0 Ω resistor are wired in series to a 12 V ideal battery. A switch in the circuit is closed at time 0, at which time the current is zero. 2.0 ms later the energy stored in the inductor is:

Final current is same as when inductor is not present $\Rightarrow i_f = \frac{12\ V}{3\ \Omega} = 4\ A$

For series LR circuit

(A)( ) 0
$\boxed{\text{(B)( ) } 1.92 \times 10^{-2}\ J}$
(C)( ) 1.1×10^{-3} J
(D)( ) 1.8×10^{-3} J
(E)( ) 2.2×10^{-3} J

$i(t) = i_f(1 - e^{-t/\tau_L})$ where $\tau_L = \frac{L}{R}$

$= i_f(1 - e^{-Rt/L}) = (4.0\ A)(1 - e^{-(3\ \Omega)(2\times10^{-3}s)/(6\times10^{-3}\ H)})$

$= 2.528\ A$

$U = \frac{1}{2}Li^2 = \frac{1}{2}(6.0\times10^{-3}\ H)(2.528\ A)^2 = 1.917\times10^{-2}\ J$

9. (10 points) An LC circuit has a capacitance of 30 μF and an inductance of 15 mH. At time $t = 0$ the charge on the capacitor is 10 μC and the current is 20 mA. The maximum current is:

initial energy $U_0 = \frac{1}{2}Li_0^2 + \frac{1}{2}\frac{q_0^2}{C}$

$=$ energy when all energy is stored in inductor

$= \frac{1}{2}Li^2$

(A)( ) 18 mA
(B)( ) 20 mA
$\boxed{\text{(C)( ) 25 mA}}$
(D)( ) 35 mA
(E)( ) 42 mA

$\Rightarrow \frac{1}{2}Li^2 = \frac{1}{2}Li_0^2 + \frac{1}{2}\frac{q_0^2}{C} \Rightarrow i^2 = i_0^2 + \frac{q_0^2}{LC}$

$\Rightarrow i = \sqrt{i_0^2 + \frac{q_0^2}{LC}} = \left((20\times10^{-3}\ A)^2 + \frac{(10\times10^{-6}\ C)^2}{(15\times10^{-3}\ H)(30\times10^{-6}\ F)}\right)^{1/2} = .0249\ A$

10. (10 points) What resistance R should be connected in series with an inductance $L =$ 220 mH and capacitance $C = 12.0\ \mu$F for the maximum charge on the capacitor to decay to 99.0% of its initial value in 50.0 cycles? (Assume $\omega' \approx \omega$.)

$q(t) = Q e^{-Rt/2L}\cos(\omega' t + \phi)$. We take $\omega' \approx \omega = \frac{1}{\sqrt{LC}}$.

(A)( ) $4.33 \times 10^{-2}\ \Omega$ Decay is due to $e^{-Rt/2L}$ term.
(B)( ) $4.33 \times 10^{-3}\ \Omega$ Find R such that $e^{-Rt/2L} = .99$
(C)( ) $6.35 \times 10^{-4}\ \Omega$ ω' has units $\frac{rad.}{sec.}$. $\omega' = 2\pi f' \Rightarrow f' = \frac{\omega'}{2\pi}\ \frac{cycles}{sec}$
$\boxed{\text{(D)( ) } 8.66 \times 10^{-3}\ \Omega}$
(E)( ) $9.57 \times 10^{-6}\ \Omega$ Period $T = \frac{1}{f'} = \frac{2\pi}{\omega'}\ \frac{sec}{cycle}$.

Time for N cycles is $t = NT = N\left(\frac{2\pi}{\omega'}\right) \approx \frac{2\pi N}{\omega} = \frac{2\pi N}{\frac{1}{\sqrt{LC}}} = 2\pi N\sqrt{LC}$

$e^{-Rt/2L} = .99 \Rightarrow e^{-\frac{R}{2L}(2\pi N\sqrt{LC})} = .99 \Rightarrow -\frac{R(2\pi N\sqrt{LC})}{2L} = \ln(.99)$

$\Rightarrow R = -\frac{\sqrt{L}\ln(.99)}{\pi N\sqrt{C}} = -\frac{\sqrt{(220\times10^{-3}\ H)}\ \ln(.99)}{\pi(50.0\ cycles)\sqrt{(12.0\times10^{-6}\ F)}} = 8.66\times10^{-3}\ \Omega$