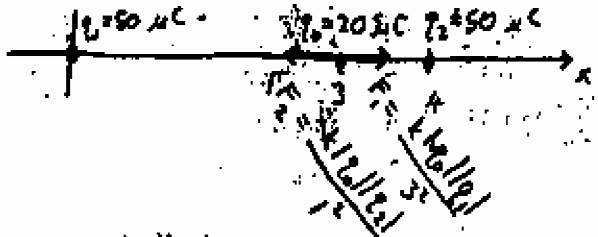
Physics 241
Final Exam
Fall 1997

1. A 50 μC charge is placed at the origin, and an identical charge is placed on the x axis at $x = 4.0$ m. What is the magnitude of the electrostatic force on a 20 μC charge placed on the x axis at $x = 3.0$ m?

- (a) 0.5 N
- [✓ (b) 8.0 N]
- (c) 9.6 N
- (d) 6.4 N
- (e) 4.8 N



$$F_x = F_1 - F_2 = \frac{k|q_0||q_1|}{1^2} - \frac{k|q_0||q_2|}{3^2} = k(20\times10^{-6}\,C)(50\times10^{-6}\,C)\left(1 - \frac{1}{3^2}\right)$$
$$= \boxed{8\ N}$$

2. A charge of 0.80 nC is placed at the center of a cube that measures 4.0 m along each edge. What is the electric flux through one face of the cube?

- (a) 64 $N \cdot m^2/C$
- (b) 23 $N \cdot m^2/C$
- (c) 45 $N \cdot m^2/C$
- [✓ (d) 15 $N \cdot m^2/C$]
- (e) 90 $N \cdot m^2/C$

Flux thru cube is

$$\Phi = \oint_{cube} \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

By symmetry, flux Φ_1 thru each face is the same, so $\Phi = 6\,\Phi_1$. Thus flux thru one face is

$$\Phi_1 = \frac{1}{6}\,\frac{q_{enclosed}}{\epsilon_0} = \frac{.8\times10^{-9}\ C}{6\left(8.85\times10^{-12}\ \frac{C^2}{N\cdot m^2}\right)} = \boxed{15.06\ \frac{N\cdot m^2}{C}}$$

3. What is the speed of a proton that has been accelerated from rest through a potential difference of 4.0 kV? (Hint: use energy)

- (a) 6.2 ×10⁵ m/s
- (b) 1.2 ×10⁶ m/s
- [✓ (c) 8.8 ×10⁵ m/s]
- (d) 9.8 ×10⁵ m/s
- (e) 1.1 ×10⁶ m/s

Energy $E = 4\times10^3\ eV \times \frac{1.6\times10^{-19}\ J}{1\ eV} = 6.4\times10^{-16}\ J = \frac{1}{2} m_p v^2$

(electron or proton accelerated thru 4×10^3 V acquires a kinetic energy of 4×10^3 electron-Volts (eV))

$$v = \sqrt{\frac{2E}{m_p}} = \left(\frac{2 \cdot 6.4\times10^{-16}\ J}{1.67\times10^{-27}\ kg}\right)^{1/2} = \boxed{8.75\times10^5\ m/s}$$

4. A capacitor of unknown capacitance C is charged to 100 V and then connected across an initially uncharged 60 μF capacitor. If the final potential difference across the 60 μF capacitor is 40 V, determine C.

- [✓ (a) 40 μF]
- (b) 90 μF
- (c) 16 μF
- (d) 32 μF
- (e) 49 μF

Initial charge on C is $q = 100\ C$ (from $q = CV$)

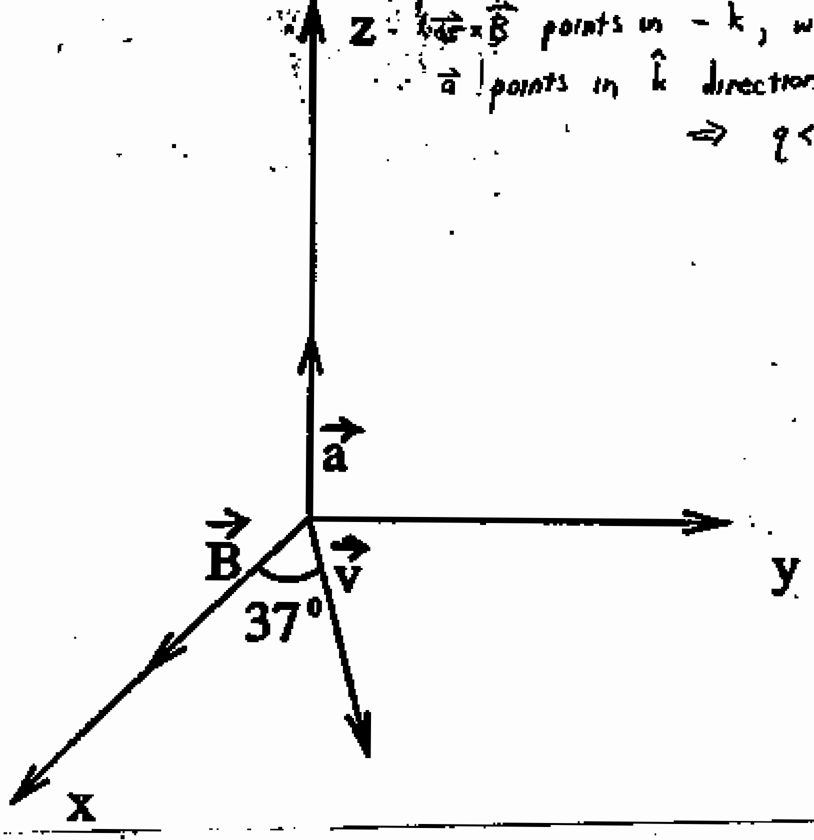
$$q = C_{12} V_{12} \Rightarrow 100\,C = (C+60)(40)$$
$$\Rightarrow 60\,C = 60 \cdot 40 \Rightarrow \boxed{C = 40\ \mu F}$$

5. A particle (mass 6.0 mg) moves in the xy plane with a speed of 4.0 km/s and a direction that makes an angle of 37° with respect to the x axis. A magnetic field of (5.0i) mT produced an acceleration of (8.0k) m/s². What is the charge of the particle?

(a) −5.0 μC
~~(b) 4.8 μC~~
$\boxed{\text{(c) } -4.0\ \mu C}$
(d) 4.0 μC
(e) −4.8 μC

$$ma = \vec F = q\,\vec v\times\vec B = qvB\sin 37°$$

$$|q| = \frac{ma}{vB\sin 37°} = \frac{(6\times10^{-6}\ kg)(8\ m/s^2)}{(4.0\times10^{3}\ m/s)(5\times10^{-3}\ T)\sin 37°} = \boxed{3.98\times10^{-6}\ C}$$



$\vec v\times\vec B$ points in $-\hat k$, whereas $\vec a$ points in $\hat k$ direction

$\Rightarrow q < 0$

6. A segment of wire of total length 2.0 m is formed into a circular loop having 5.0 turns. If the wire carries a 1.2 A current, determine the magnitude of the magnetic field at the center of the loop.

(a) 9.4 μT
(b) 89 μT
$\boxed{\text{(c) } 59\ \mu T}$
(d) 69 μT
(e) 79 μT

perimeter: $2\pi R = \frac{2\ m}{5} \Rightarrow R = \frac{2\ m}{2\pi(5)}$

$$d\vec B = \frac{\mu_0 i}{4\pi}\frac{d\vec s\times\hat r}{r^2} = \frac{\mu_0 i}{4\pi}\frac{ds}{r^2} \quad \text{since } \hat r \perp d\vec s$$

B-field due to 1 turn:

$$B_1 = \int_{circle} \frac{\mu_0 i\,ds}{4\pi r^2} = \frac{\mu_0 i}{4\pi R^2}\underbrace{\int_{circle} ds}_{2\pi R} = \frac{\mu_0 i}{2R}$$

B-field due to $N=5$ turns:

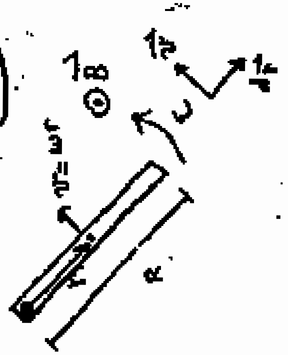
$$B = NB_1 = \frac{\mu_0 i N}{2R} = \frac{(4\pi\times10^{-7}\ \frac{T\cdot m}{A})(1.2\ A)(5)}{2\left(\frac{2}{2\pi(5)}\right)} = \boxed{59.2\times10^{-6}}$$

7. A conducting rod (length = 80 cm) rotates at a constant angular rate of 15 revolutions per s about a pivot at one end. A uniform field (B = 60 mT) is directed perpendicularly to the plane of rotation. What is the magnitude of the emf induced between the ends of the rod?

(a) 3.3 V
$\boxed{\text{(b) } 1.8\ V}$
(c) 2.4 V
(d) 2.1 V
(e) 2.7 V

Force on charge in rod in equilibrium is zero, otherwise charge would be moving. ∴ $\vec F = q(\vec E + \vec v\times\vec B) = \vec 0$

$$\Rightarrow \underbrace{-\vec E = \vec v\times\vec B}$$



$$V(R) - V(0) = -\int_0^R \vec E\cdot d\vec r = \int_0^R (\vec v\times\vec B)\cdot d\vec r$$

$$= \int_0^R vB\,dr \quad \text{since } \vec v\times\vec B \parallel d\vec r \text{ and } \vec v\perp\vec B$$

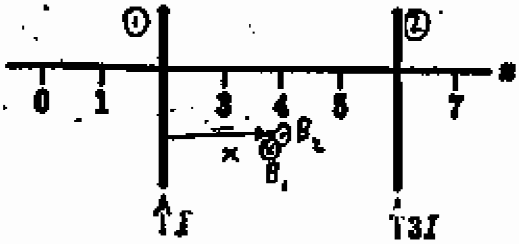
$$= \int_0^R r\omega B\,dr = \omega B\left[\tfrac{1}{2}r^2\right]_0^R$$

$$= \tfrac{1}{2}\omega BR^2 = \tfrac{1}{2}(2\pi f)BR^2$$

$$= \pi(15\ Hz)(60\times10^{-3}\ T)(.8\ m)^2 = \boxed{1.809\ V}$$

8. Two long straight current-carrying parallel wires cross the x axis and carry currents I and $3I$ in the same direction, as shown. At what value of x is the net magnetic field zero?

(a) 7
(b) 5
[✓ (c) 3]
(d) 1
(e) 0



$\vec{B}$-field at a distance r from wire is

$$|\vec{B}| = \frac{\mu_0 i}{2\pi r}$$

Component of B into page a distance r from wire ① is

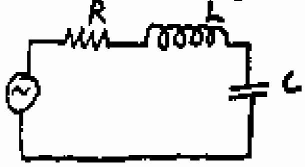
$$\frac{\mu_0 I}{2\pi r} - \frac{\mu_0 3I}{2\pi(4-r)} = 0$$

$$\therefore \frac{1}{r} = \frac{3}{4-r} \Rightarrow 4-r=3r \Rightarrow 4r=4$$

$$\Rightarrow r=1 \Rightarrow x=r+2=1+2=\boxed{3}$$

9. What is the average power dissipation in an RLC series circuit $R = 10\Omega$, L = 0.1 H, C = 10 μF driven at resonance by a 100 V rms source?

(a) 700 W
(b) 2 kW
(c) 100 W
(d) 500 W
[✓ (e) 1000 W]



$$|Z| = \sqrt{R^2+(X_L-X_C)^2} \qquad X_L-X_C = \omega L - \frac{1}{\omega C}$$

At resonance, $X_L - X_C = 0$, so $|Z| = R$

$$i_{rms} = \frac{V_{rms}}{|Z|} = \frac{V_{rms}}{R}$$

$$\bar{P} = i_{rms}^2 R = \left(\frac{V_{rms}}{R}\right)^2 R = \frac{V_{rms}^2}{R} = \frac{(100\ V)^2}{10} = \boxed{1000\ W}$$

10. If the maximum E-component of an electromagnetic wave is 600 V/m, what is the maximum B-component?

[✓ (a) 2×10^{-6} T]
(b) 10^{-3} T
(c) 1.6×10^{-10} T
(d) 1.8×10^{-8} T
(e) 1.4 Tesla

$$\frac{E_m}{B_m} = c \Rightarrow B_m = \frac{E_m}{c} = \frac{600\ V/m}{3\times10^8\ m/s} = \boxed{2\times10^{-6}\ T}$$

11. A light ray whose frequency is 6×10^{14} Hz in vacuum is incident on water (n = 1.33). The wavelength of the light after it enters the water is (in nm)

(a) 266
(b) 798
(c) 665
(d) 500
[✓ (e) 376]

$f = 6\times10^{14}$ Hz $\qquad \lambda f$ = speed of wave, in vacuum = c $\Rightarrow \lambda = \frac{c}{f}$

$$\lambda_n = \frac{\lambda}{n} = \frac{c}{nf} = \frac{3\times10^8\ m/s}{(1.33)(6\times10^{14}\ Hz)} = \boxed{375.9\ nm}$$

12. Two mirrors are at right angles to one another. A light ray is incident on the first at an angle of 30° with respect to the normal to the surface. What is the angle of reflection with respect to the normal to the second surface?

(a) 75°
(b) 53°
(c) 45°
[✓ (d) 60°]
(e) 30°

13. **A diver shines an underwater searchlight at the surface of a pond ($n_w = 1.33$). Below, what angle (relative to the surface) will the light be totally reflected?**

(a) 69°
(b) 58°
(c) 51°
$\boxed{\text{(d) 41°}}$
(e) 47°

$n_a \sin 90° = n_w \sin \theta_c$

$\theta_c = \sin^{-1}\left(\frac{n_a}{n_w}\right) = 48.75$

$\theta = 90 - \theta_c = 41.25°$

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14. **A convex mirror has a focal length of − 20 cm. What it the position of the resulting image (in cm) if the image is erect and four times smaller than the object?**

(a) − 10 cm
$\boxed{\text{(b) − 15}}$ cm
(c) − 50 cm
(d) − 25 cm
(e) − 100 cm

$f = -20$ cm $\qquad m = -\frac{i}{p} = \frac{1}{4} \Rightarrow p = -4i$

$\frac{1}{f} = \frac{1}{i} + \frac{1}{p} = \frac{1}{i} + \frac{1}{-4i} = \frac{1}{i}(1 - \frac{1}{4}) = \frac{3}{4i} \Rightarrow i = \frac{3f}{4} = \frac{3(-20\text{ cm})}{4} = \boxed{-15\text{ cm}}$

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15. **A customer stands 5 m in front of a concave mirror with a radius of curvature of 100 cm. What is the size of his image in the mirror if he is 2 m tall?**

$\boxed{\text{(a) 22}}$
(b) 88
(c) 18
(d) 48
(e) 68

Concave $\Rightarrow f > 0 \qquad f = \frac{r}{2} = \frac{100\text{ cm}}{2} = 50$ cm $\qquad p = 500$ cm

$\frac{1}{i} + \frac{1}{p} = \frac{1}{f} \Rightarrow i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{50} - \frac{1}{500}} = 55.5$ cm

$m = -\frac{i}{p} \Rightarrow \frac{h'}{h} = |m| = \frac{|i|}{|p|} \Rightarrow h' = \frac{|i|}{|p|}h = \frac{55.5\text{ cm}}{500\text{ cm}}(200\text{ cm}) = \boxed{22.2\text{ cm}}$

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16. **An object 20 cm high is placed 2 m in front of a lens whose focal length is +5.0 cm. What is the size of the image (in mm)?**

$f = +5.0$ cm

(a) 3
$\boxed{\text{(b) 5}}$
(c) 7
(d) 6
(e) 8

$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{5} - \frac{1}{200}} = 5.12$ cm

$m = -\frac{i}{p} \Rightarrow |m| = \frac{h'}{h} = \left|\frac{i}{p}\right| \Rightarrow h' = \frac{|i|}{p}h = \frac{5.12\text{ cm}}{200\text{ cm}} 20\text{ cm} = \boxed{5\text{ mm}}$

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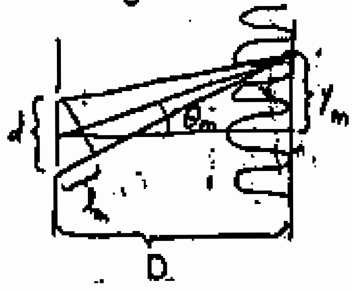
17. **A laser beam ($\lambda = 694$ nm) is incident on two slits 0.1 mm apart. Approximately how far apart (in m) will the bright interference fringes be on the screen 5 m from the double slits?**

(a) 3.47×10^{-5} m
(b) 3.47×10^{-6} m
(c) 3.47×10^{-4} m
$\boxed{\text{(d) } 3.47 \times 10^{-2}}$ m
(e) 3.47×10^{-3} m



m^{th} maximum occurs when

$d \sin \theta_m = m\lambda$

$\tan \theta_m = \frac{y_m}{D}$

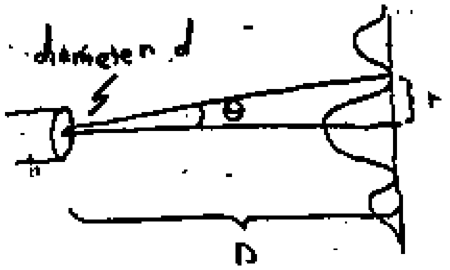
small angle approx $\Rightarrow \sin \theta_m \approx \tan \theta_m$

$\Rightarrow \frac{m\lambda}{d} = \sin \theta_m \approx \tan \theta_m = \frac{y_m}{D}$

$\therefore y_{m+1} - y_m = \frac{D(m+1)\lambda}{d} - \frac{Dm\lambda}{d} = \frac{D\lambda}{d} = \frac{(5\text{ m})(694 \times 10^{-9}\text{ m})}{.1 \times 10^{-3}\text{ m}} = \boxed{3.47 \times 10^{-2}\text{ m}}$

18. A ruby laser beam ($\lambda = 694.3$ nm) is sent outwards from a 2.7 m diameter telescope to the moon, 384,000 km away. What is the radius of the big red spot on the moon?

(a) 2.7 km
(b) 1 km
(c) 500 m
(d) 250 m
[✓ (e) 120 m]



$\sin\theta = \frac{1.22\lambda}{d}$

$\tan\theta = \frac{r}{D}$

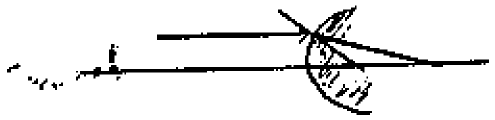
θ small $\Rightarrow \sin\theta \approx \tan\theta$

$\Rightarrow \frac{1.22\lambda}{d} \approx \frac{r}{D}$

$$\therefore\ r = \frac{D \cdot 1.22\lambda}{d} = \frac{(384\,000\times10^{3}\ m)1.22(694.3\times10^{-9}\ m)}{2.7\ m} = \boxed{120.46\ m}$$

19. A convex refracting surface has a radius of 12 cm. Light is incident in air ($n = 1$) and is refracted into a medium with an index of refraction of 2. Light incident parallel to the central axis is focused at a point:

[✓ (a) 24 cm from the surface]
(b) 18 cm from the surface
(c) 12 cm from the surface
(d) 6 cm from the surface
(e) 3 cm from the surface



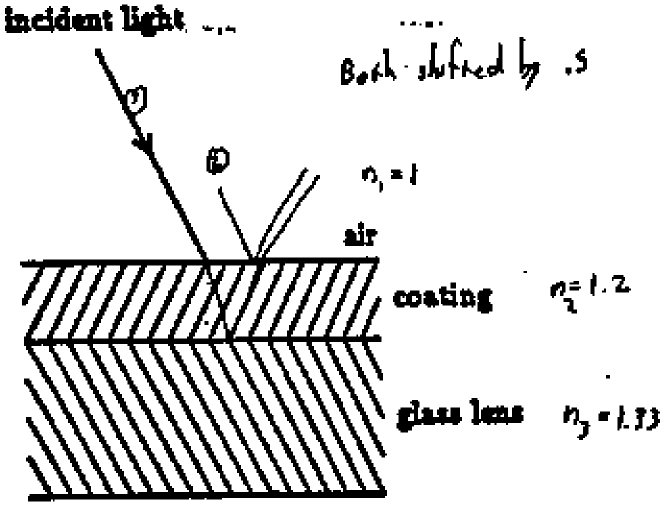
$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{R}$ $\quad p = \infty \Rightarrow \quad \frac{2}{i} = \frac{2-1}{R} = \frac{1}{R} \Rightarrow i = 2R = \boxed{24\ cm}$

20. A lens is coated with a material of index of refraction 1.2 in order to minimize reflection. If λ denotes the wavelength of the incident light in air, what is the thinnest possible such coating?

(a) 0.25 λ
[✓ (b) 0.208 λ]
(c) 0.3 λ
(d) 0.416 λ
(e) 0.5 λ

$\frac{2\ell_m}{\lambda_{n_2}} = \frac{1}{2}$

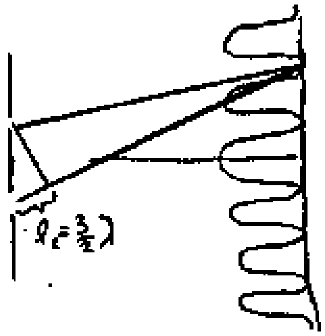
$\ell_m = \frac{1}{4}\frac{\lambda}{n_2} = \frac{1}{4(1.2)}\lambda = \boxed{.208\ \lambda}$



Both shifted by .5

21. Waves from two slits are in phase at the slits and travel to a distant screen to produce the second minimum of the interference pattern. The difference in the distance traveled by the waves is:

(a) five halves of a wavelength
(b) two wavelengths
[✓ (c) three halves of a wavelength]
(d) a wavelength
(e) half a wavelength



minima occur when path length diff is $\ell_m = (m - \frac{1}{2})\lambda$

$m = 2 \Rightarrow \ell_m = \boxed{\frac{3}{2}\lambda}$