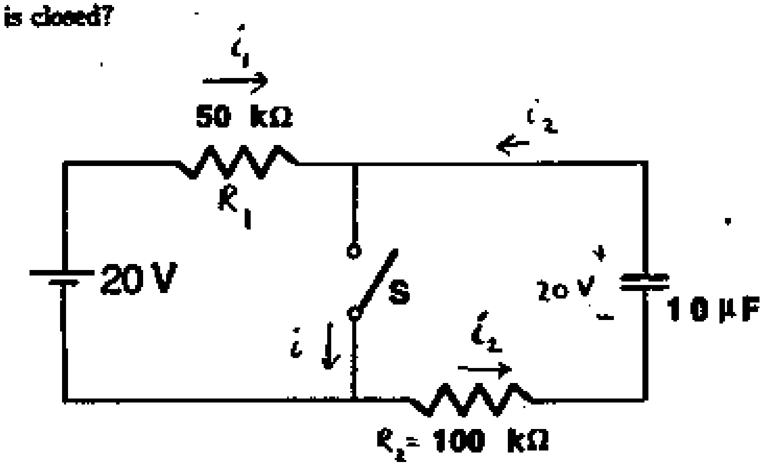
(Phys 241 Final May 1997) (KEY)

1. (8 points) In the circuit shown below the switch S has been open for a long time. It is then suddenly closed. What is the current through the switch immediately when it is closed?



(A)( ) 133 μA up
(B)( ) 133 μA down
(C)( ) 600 μA up
$\boxed{\text{(D)( ) 600 μA down}}$
(E)( ) None of the above

Capacitor charged to 20 V. When switch closes, 20 V across R_1

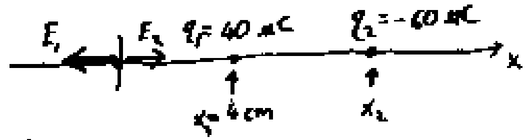
$\Rightarrow i_1 = \frac{20\ V}{50 \times 10^3\ \Omega}$

20 V across $R_2 \Rightarrow i_2 = \frac{20\ V}{100 \times 10^3\ \Omega}$

$i = i_1 + i_2 = \frac{20\ V}{50 \times 10^3\ \Omega} + \frac{20\ V}{100 \times 10^3\ \Omega} = 600\ \mu A$ down

2. (8 points) A 40 μC charge is located on the x axis at $x = 4.0$ cm. At what x should a −60 μC charge be placed to produce a net electric field of zero at the origin ($x = 0$)?

(A)( ) −5.3 cm
(B)( ) +5.7 cm
$\boxed{\text{(C)( ) +4.9 cm}}$
(D)( ) −6.0 cm
(E)( ) +6.0 cm



Field due to $q_1 = 40\ \mu C$ points in $-x$ direction at origin; field due to $q_2 = -60\ \mu C$ must point in $+x$ to get cancellation; $\Rightarrow x_2 > 0$.

$E_x = E_1 + E_2 = -\frac{k|q_1|}{x_1^2} + \frac{k|q_2|}{x_2^2} = 0$

$\Rightarrow x_2 = x_1 \sqrt{\left|\frac{q_2}{q_1}\right|} = (4\ cm)\sqrt{\left|\frac{-60\ \mu C}{40\ \mu C}\right|} = +4.9\ cm$

3. (8 points) A wire of diameter D, length L, and resistance R is replaced in a circuit by a wire of the same material having a diameter $D/2$ and length ~~$L/2$~~ $2L$. The resistance of the new wire is:

(A)( ) 16R
$\boxed{\text{(B)( ) 8R}}$
(C)( ) 2R
(D)( ) R
(E)( ) R/2

$R = \rho \frac{\text{length}}{\text{area}} = \rho \frac{L}{\pi\left(\frac{D}{2}\right)^2}$ (ρ = resistivity)

$R' = \rho \frac{2L}{\pi\left(\frac{D}{4}\right)^2} = 8\rho \frac{L}{\pi\left(\frac{D}{2}\right)^2} = 8R$

4. (8 points) The minimum capacitance that one can obtain by connecting together (either in series, or parallel or a mixed combination) three capacitors of values 1.0 μF, 2.0 μF, and 3.0 μF is:

$\boxed{\text{(A)( ) 6/11 μF}}$
(B)( ) 7/13 μF
(C)( ) 5/12 μF
(D)( ) 6/5 μF
(E)( ) 3/8 μF

Capacitors in parallel add. For minimum capacitance put them in series:

$\frac{1}{C} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6} \Rightarrow C = \frac{6}{11}\ \mu F$

5. (8 points) The magnetic flux through a coil of resistance 30 Ω is changed from $2 \times 10^{-5}\ T \cdot m^2$ to $5 \times 10^{-5}\ T \cdot m^2$ in 0.1 second. How much charge will flow through the coil during this time?

Take N (# turns) = 1

(A)( ) 1×10^{-4} ~~μ~~C
(B)( ) 2×10^{-6} ~~μ~~C
(C)( ) 3×10^{-6} ~~μ~~C
$\boxed{\text{(D)( ) } 1 \times 10^{-6}\text{ C}}$
(E)( ) 1×10^{-7} ~~μ~~C

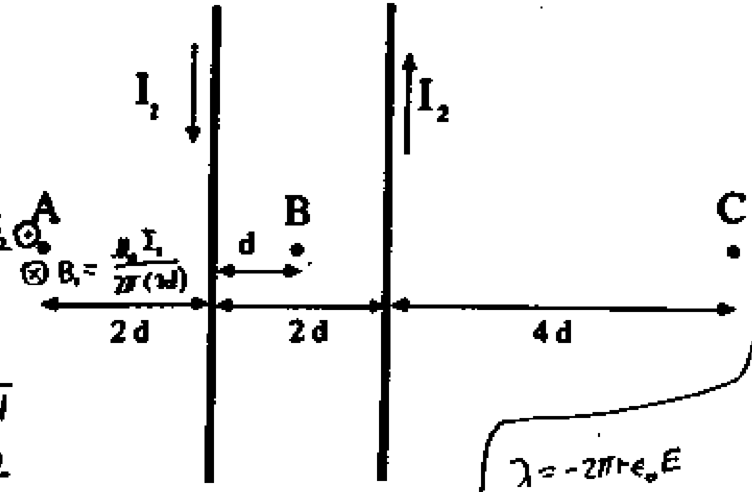
$|\mathcal{E}| = \left|N \frac{\Delta\Phi}{\Delta t}\right| = \left|\frac{\Delta\Phi}{\Delta t}\right|$

current $|i| = \frac{|\mathcal{E}|}{R} = \frac{1}{R}\left|\frac{\Delta\Phi}{\Delta t}\right|$

$|i| = \left|\frac{\Delta q}{\Delta t}\right| = \frac{1}{R}\left|\frac{\Delta\Phi}{\Delta t}\right|$

$\Rightarrow |\Delta q| = \frac{|\Delta\Phi|}{R} = \frac{5 \times 10^{-5} - 2 \times 10^{-5}}{30} = 1 \times 10^{-6}\ C$

6. (8 points) Two straight, very long, parallel conductors carry currents I_1 and I_2 in the directions as shown in the figure below. If the magnetic field at point A is to be zero, the ratio of the currents I_2/I_1 must be:



$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enc}$

$2\pi r \, B = \mu_0 i_{enc}$

$B = \frac{\mu_0 i_{enc}}{2\pi r}$

$\frac{\mu_0 I_2}{2\pi(2d+2d)} = B_2 \odot \quad \otimes B_1 = \frac{\mu_0 I_1}{2\pi(2d)}$

$\therefore \frac{\mu_0 I_2}{2\pi \cdot 4d} = \frac{\mu_0 I_1}{2\pi \cdot 2d}$

$\Rightarrow \frac{I_2}{I_1} = 2$

(A)( ) 1.5
$\boxed{\text{(B)( ) 2.0}}$
(C)( ) 2.5
(D)( ) 3.0
(E)( ) At C magnetic field can not be zero.

7. (8 points) An electron follows a circular path (of radius $r = 15$ cm) in a uniform magnetic field $B = 3 \times 10^{-4}$ T. What is the period of the circular motion?

$\boxed{\text{(A)( ) 0.12 μs}}$
(B)( ) 1.2 ms
(C)( ) 0.18 μs
(D)( ) 1.8 ms
(E)( ) 1.8 μs

$ma = F_B \Rightarrow \frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$

period $T = \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$

$= \frac{2\pi (9.11\times10^{-31}\ kg)}{(1.60\times10^{-19} C)(3\times10^{-4} T)} = .119\ \mu s$

8. (8 points) A sinusoidal voltage $V(t) = (40\text{ V})\sin(100t)$ is applied to a series LCR circuit with $L = 200$ mH, $C = 50\ \mu$F, and $R = 120\ \Omega$. The phase angle ϕ between the current and the voltage is:

$\boxed{\text{(A)( ) −56.3°}}$
(B)( ) 56.3°
(C)( ) −42.1°
(D)( ) 42.1°
(E)( ) 0°

$\omega = 100$ rad/s

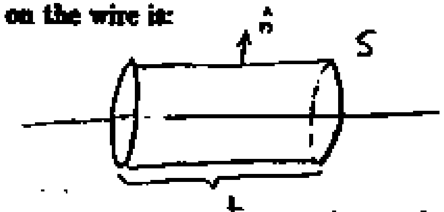
$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

$\phi = \tan^{-1}\left(\frac{(100 \frac{rad}{s})(200\times10^{-3} H) - \frac{1}{(100 \frac{rad}{s})(50\times10^{-6} F)}}{120\ \Omega}\right)$

$= -56.3°$

9. (8 points) A very long straight wire has a charge uniformly distributed over its length. The electric field at a distance of 10 cm from the wire has a magnitude of 1.62×10^7 N/C and is directed __toward__ the wire and is perpendicular to the wire. The linear charge density on the wire is:

(A)( ) −75 μC/m
(B)( ) +75 μC/m
$\boxed{\text{(C)( ) −90 μC/m}}$
(D)( ) +90 μC/m
(E)( ) None of the above



Let the Gaussian surface S be a cylinder of length L and radius r = 10 cm.

$\oint_S \vec{E} \cdot d\vec{A} = \frac{\text{charge in S}}{\epsilon_0} \Rightarrow E \cdot 2\pi r L = \frac{-\lambda L}{\epsilon_0} \Rightarrow \lambda = -2\pi r \epsilon_0 E$

$= -2\pi($

$\lambda = -2\pi r \epsilon_0 E$

$= -2\pi(.10 m)(8.85\times10^{-12})\times(1.62\times10^7)$

$= -90\ \mu C/m$

10. (8 points) The potential difference V between the two plates of a parallel-plate capacitor is maintained at a constant value by connecting it to a battery. If the plates are pulled apart (maintaining the constant potential difference between the plates) how does the magnitude of the charge on the plates change?

(A)( ) Increases
$\boxed{\text{(B)( ) Decreases}}$
(C)( ) Does not change
(D)( ) Depends on V
(E)( ) Depends on the area of the plates

For parallel plate capacitor

$C = \frac{K\epsilon_0 A}{d}$

$d \uparrow \Rightarrow C \downarrow$

$\Rightarrow q = CV \downarrow$

since V is held constant

11. (8 points) When the switch S in the circuit shown below is thrown closed, the current takes 3.00 ms to reach 95 % of its final value. If $R = 20.0\ \Omega$, what is the value of the inductance L?

$i = \underbrace{\frac{E}{R}}_{i_o}\left(1 - e^{-t/\tau_L}\right)$

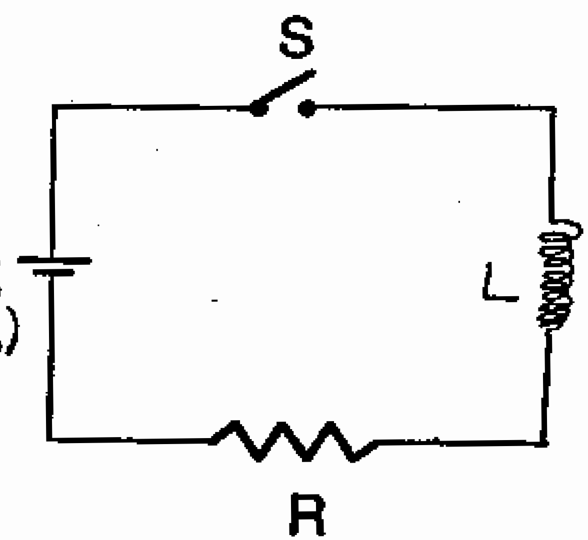
$e^{-t/\tau_L} = 1 - \frac{i}{i_o}$

$-\frac{t}{\tau_L} = \ln\left(1 - \frac{i}{i_o}\right)$

$t = -\tau_L \ln\left(1 - \frac{i}{i_o}\right) = -\frac{L}{R}\ln\left(1 - i/i_o\right)$

$L = -\frac{Rt}{\ln\left(1-\frac{i}{i_o}\right)} = -\frac{(20\ \Omega)(3\times10^{-3}s)}{\ln(1-.95)}$

$= 20\times10^{-3}\ H$



(A)( ) 10 mH
(B)( ) 15 mH
(C)( ) 20 mH
(D)( ) 25 mH
(E)( ) 30 mH

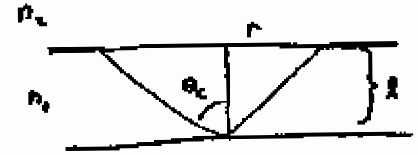
12. (8 points) A wire carries a steady current of 2.5 A. A straight portion of the wire is 1.00 m long and lies along the x axis within a magnetic field $\mathbf{B} = (1.6\hat{k})$ T. If the current is in the $+x$ direction, the magnetic force on the section of the wire is:

$\vec{F} = i\vec{L}\times\vec{B} = (2.5\ A)(1.00\ \hat{i}\ m)\times(1.6\ \hat{k}\ T)$

$= 4\ \underbrace{\hat{i}\times\hat{k}}_{-\hat{j}}\ N = -4\hat{j}\ N$

(A)( ) $(+6.0\hat{k})$ N
(B)( ) $(-6.0\hat{k})$ N
(C)( ) $(+4.0\hat{j})$ N
(D)( ) $(-4.0\hat{j})$ N
(E)( ) $(-2.0\hat{i})$ N

13. (8 points) A bright point source of light is placed at the bottom of a large swimming pool. If the depth of the water in the pool is 2.0 m, what is the radius of the largest circle at the surface of water through which light can emerge?



(A)( ) 2.28 m
(B)( ) 3.15 m
(C)( ) 4.22 m
(D)( ) 1.96 m
(E)( ) 0.84 m

$n_1 \sin\theta_c = n_2 \sin 90 = 1 \qquad \sin\theta_c = \frac{1}{1.33}$

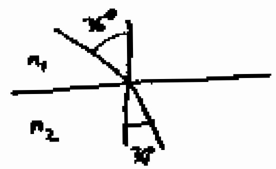
$\theta_c = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75°$

$r = \ell \tan\theta_c = (2.0\ m)\tan(48.75°) = 2.28\ m$

14. (8 points) A light beam in air is incident on a piece of glass. Which of the properties of the light are changed upon entering into the glass?

(A)( ) the frequency, velocity, and wavelength
(B)( ) the frequency and the velocity
(C)( ) the velocity and the wavelength
(D)( ) the velocity only
(E)( ) the frequency and the wavelength

15. (8 points) A light ray initially in air enters a transparent plastic material at an angle of incidence of 46° 36° and the transmitted ray (inside the plastic) is refracted at an angle of 34°. What is the speed of light in the plastic material?



(A)( ) 1.22×10^8 m/s
(B)( ) 1.37×10^8 m/s
(C)( ) 1.75×10^8 m/s
(D)( ) 2.14×10^8 m/s
(E)( ) 2.86×10^8 m/s

$n_1 \sin 36° = n_2 \sin 34°$

$\Rightarrow n_2 = \frac{(1)\sin 36°}{\sin 34°} = 1.0511$

$v = \frac{c}{n_2} = \frac{3\times10^8\ m/s}{1.0511} = 2.854\times10^8\ m/s$

21. (8 points) The electric field in an electromagnetic wave (in SI units) is described by the equation:

$$\mathbf{E} = \hat{y}(180)\sin(5.0\times 10^{6}z - \omega t)$$

The magnitude and direction of the corresponding magnetic field is:

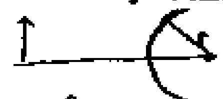
$\boxed{\text{(A)(\ \ )}\ 6.0\times 10^{-7}\ \text{T},\ \hat{z}}$
(B)( ) 6.4×10^{-6} T, $-\hat{z}$
(C)( ) 6.4×10^{-6} T, $\hat{x}$
(D)( ) 6.0×10^{-7} T, $-\hat{x}$
(E)( ) 5.6×10^{-7} T, $-\hat{y}$

Propagating in $+x$ direction
(for larger t, x must increase to keep argument same
$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ $\quad \vec{S}$ in $\hat{x}$ direction, $\vec{E}$ in $\hat{y}$ direction

$B = \frac{E}{c} = \frac{180\ \text{V/m}}{3\times 10^{8}\ \text{m/s}} = 6.0\times 10^{-7}\ \text{T}$ $\quad \vec{B}$ in $+\hat{z}$ direction

22. (8 points) You are standing 50 cm in front of a reflective spherical Christmas-tree ornament 8 cm in diameter. The location of your image is:

(A)( ) 2.08 cm, in front
(B)( ) 2.08 cm, behind
(C)( ) 1.83 cm, in front
(D)( ) 1.83 cm, behind
(E)( ) 4.00 cm, in front

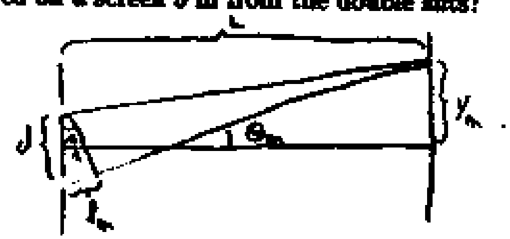


radius of curvature $= \frac{8\text{ cm}}{2} = 4$ cm
convex $\Rightarrow r<0 \Rightarrow r=-4$ cm
$\Rightarrow f = \frac{r}{2} = -2$ cm

$\frac{1}{i}+\frac{1}{p}=\frac{1}{f} \Rightarrow i = \frac{1}{\frac{1}{f}-\frac{1}{p}} = \frac{1}{\frac{1}{-2}-\frac{1}{50}} = -1.92$ cm $\quad \boxed{1.92\text{ cm behind}}$

23. (8 points) A laser beam with wavelength $\lambda = 471$ nm is incident upon two slits 0.2 mm apart. How far will the third dark fringe be from the central maximum, if the interference pattern is observed on a screen 5 m from the double slits?

(A)( ) 41.2 mm
(B)( ) 37.7 mm
(C)( ) 23.6 mm
(D)( ) 18.5 mm
(E)( ) 12.8 mm



destructive interference when $\ell_m = (m-\frac{1}{2})\lambda = d\sin\theta_m$
3rd dark fringe corresponds to $m=3$ in above. $\theta_3 = \sin^{-1}\left(\frac{(3-\frac{1}{2})\lambda}{d}\right) = \sin^{-1}\left(\frac{5/2\,(471\times 10^{-9}\text{ m})}{.2\times 10^{-3}\text{ m}}\right) = .337°$
$\tan\theta_3 = \frac{y_3}{L} \Rightarrow y_3 = L\tan\theta_3 = (5\text{ m})\cdot\tan(.337°) = \boxed{29.4\text{ mm}}$
Also could have used $\frac{(m-\frac{1}{2})\lambda}{d} = \sin\theta_m \approx \tan\theta_m = \frac{y_m}{L} \Rightarrow y_m = \frac{L(m-\frac{1}{2})\lambda}{d}$

24. (8 points) Two spectral lines in the sodium yellow doublet have the wavelengths of 588.995 nm and 589.592 nm respectively. What is the minimum number of lines a diffraction grating must have to resolve these two lines in first order?

(A)( ) 759
$\boxed{\text{(B)(\ \ )}\ 987}$
(C)( ) 1326
(D)( ) 2529
(E)( ) 3662

required resolving power to resolve two lines $= R = \frac{\lambda_{ave}}{\Delta\lambda} = \frac{\frac{588.995+589.592}{2}}{589.592-588.995} = 987.09$
resolving power for grating is $R = Nm$.
To resolve to first order $(m=1)$, have $R = N(1) = N$,
so minimum # of lines is $N = 987$

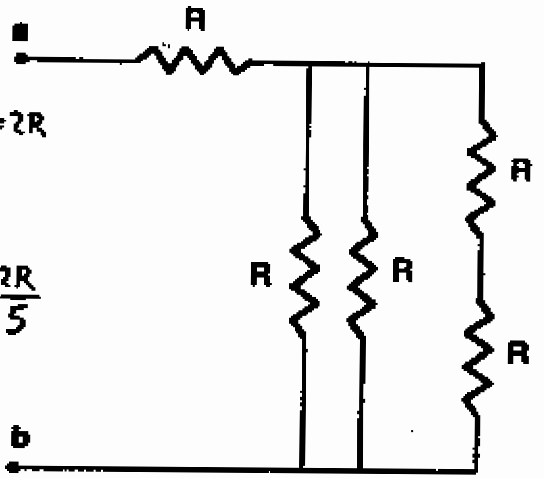
25. (8 points) A person looks at a gem through a converging lens with a focal length of 12.5 cm. A virtual image of the gem is formed 30.0 cm from the lens. The magnification of the lens is:

(A)( ) 2.61
(B)( ) −2.61
$\boxed{\text{(C)(\ \ )}\ 3.40}$
(D)( ) −3.40
(E)( ) +0.85

virtual image $\Rightarrow i = -30$ cm
converging $\Rightarrow f>0 \Rightarrow f = 12.5$ cm
$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \Rightarrow p = \frac{1}{\frac{1}{f}-\frac{1}{i}} = \frac{1}{\frac{1}{12.5}-\frac{1}{-30}}$ cm $= 8.823$ cm

$m = -\frac{i}{p} = -\frac{-30\text{ cm}}{8.823\text{ m}} = +3.4$

16. (8 points) Five identical resistors, each of value R, are connected as shown in the figure below. If the resistance between the terminals a and b is 140 Ω, what is the value of R?



R

$\frac{1}{\frac{1}{R}+\frac{1}{R}} = \frac{R}{2}$ $\quad R+R=2R$

R

$\frac{1}{\frac{1}{R/2}+\frac{1}{2R}} = \frac{1}{5/2R} = \frac{2R}{5}$

$R+\frac{2R}{5} = \frac{7R}{5} = 140\ \Omega$

$\Rightarrow R = 100\ \Omega$

(A)( ) 80 Ω
$\boxed{\text{(B)( ) 100 Ω}}$
(C)( ) 140 Ω
(D)( ) 210 Ω
(E)( ) 280 Ω

17. (8 points) Niobium metal becomes superconducting when cooled below 9 K. If the superconductivity is destroyed when the surface magnetic field exceeds 0.10 T, determine the maximum current a 2.4-mm-diameter wire can carry and remain superconducting.

(A)( ) 120 A
(B)( ) 210 A
(C)( ) 300 A
(D)( ) 480 A
$\boxed{\text{(E)( ) 600 A}}$

$r = \frac{2.4\text{ mm}}{2} = 1.2\text{ mm}$

$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 i$

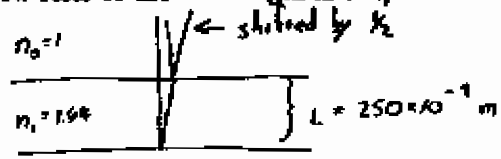
$B \cdot 2\pi r = \mu_0 i$

$B = \frac{\mu_0 i}{2\pi r} < B_{max}$

$\Rightarrow i < \frac{2\pi r B_{max}}{\mu_0} = \frac{2\pi(1.2\times10^{-3}\text{ m})(.10\text{ T})}{4\pi\times10^{-7}}$

18. (8 points) Sunlight falls on a thin oil film (index of refraction $n = 1.44$) which is floating on water (index of refraction $n = 1.33$). The thickness of the film is 250 nm. What color does this film appear when viewed from directly above? (The numbers in the parentheses below refer to the wavelength in air.)

(A)( ) Yellow (600 nm)
(B)( ) Green (532 nm)
$\boxed{\text{(C)( ) Blue (480 nm)}}$
(D)( ) Violet (400 nm)
(E)( ) Orange (674 nm)



$2L = (m+\frac{1}{2})\lambda_1 = (m+\frac{1}{2})\frac{\lambda}{n_1}$

$\lambda = \frac{2Ln_1}{m+\frac{1}{2}} = \frac{2(250\text{ nm})(1.44)}{m+\frac{1}{2}}$

$m=0 \Rightarrow \lambda = 1440$ nm (outside visible)

$m=1 \Rightarrow \lambda = 480$ nm

19. (8 points) The single slit diffraction pattern produced by a slit 2.5×10^{-4} m in width, illuminated by the light of wavelength 625 nm is observed on a screen. If the distance between the first and third minima in the diffraction pattern is 3.0 mm, how far is the screen from the slit?

$\boxed{\text{(A)( ) 60 cm}}$
(B)( ) 72 cm
(C)( ) 84 cm
(D)( ) 96 cm
(E)( ) 116 cm

a = width of slit. Assume θ_m small, so $\sin\theta_m \approx \tan\theta_m$

$a\sin\theta_1 = \lambda$ $\quad a\sin\theta_3 = 3\lambda$

$a\tan\theta_1 = \lambda$ $\quad a\tan\theta_3 = 3\lambda$

$a\frac{Y_1}{D} = \lambda$ $\quad a\frac{Y_3}{D} = 3\lambda$

$Y_3 - Y_1 = \frac{3D\lambda}{a} - \frac{D\lambda}{a} = \frac{2D\lambda}{a}$

$\Rightarrow D = \frac{a(Y_3-Y_1)}{2\lambda} = \frac{(2.5\times10^{-4}\text{ m})(3.0\times10^{-3}\text{ m})}{2(625\times10^{-9}\text{ m})} = \boxed{.6\text{ m}}$

20. (8 points) Light containing colors red, yellow, violet, blue, and green falls on a diffraction grating. Which color line appears closest to the central spot?

(A)( ) yellow
(B)( ) green
(C)( ) blue
$\boxed{\text{(D)( ) violet}}$
(E)( ) red

For a diffraction grating, angle θ_1 that 1st order maximum makes with central axis satisfies $d\sin\theta_1 = m\lambda = \lambda$

d = grating spacing (fixed)

$\lambda = d\sin\theta_1$, so as $\lambda\uparrow$, $\sin\theta_1\uparrow$ and thus $\theta_1\uparrow$

∴ line closest to central spot (smallest θ_1) corresponds to smallest wavelength λ, which is violet.