Final Exam PHYS-241
December 16, 2004

1.- Three 8 1/2" x 11" crib sheets are allowed. They must be of your own creation.
2.- Please print your name on the top edge of the op-scan sheet and sign it.
3.- Use a #2 pencil to fill in your full name, your student identification number,your recitation division number, and finally the answers for problems 1–20.

$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \, \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}$$

$$\varepsilon_0 = 8.85\times10^{-12} \, \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$$

$$\mu_0 = 4\pi\times10^{-7} \, \frac{\mathrm{N}}{\mathrm{A^2}}$$

$$e = 1.602\times10^{-19}\,\mathrm{C}$$

$c = 2.99792458\times10^8$ m/s (speed of light)

$N_{Avogadro} = 6.022\times10^{23}$ (number of atoms in 12 g of ^{12}C)

$\mathrm{m} \Rightarrow 10^{-3}$ $\mu \Rightarrow 10^{-6}$ $\mathrm{n} \Rightarrow 10^{-9}$ $\mathrm{p} \Rightarrow 10^{-12}$ $\mathrm{f} \Rightarrow 10^{-15}$

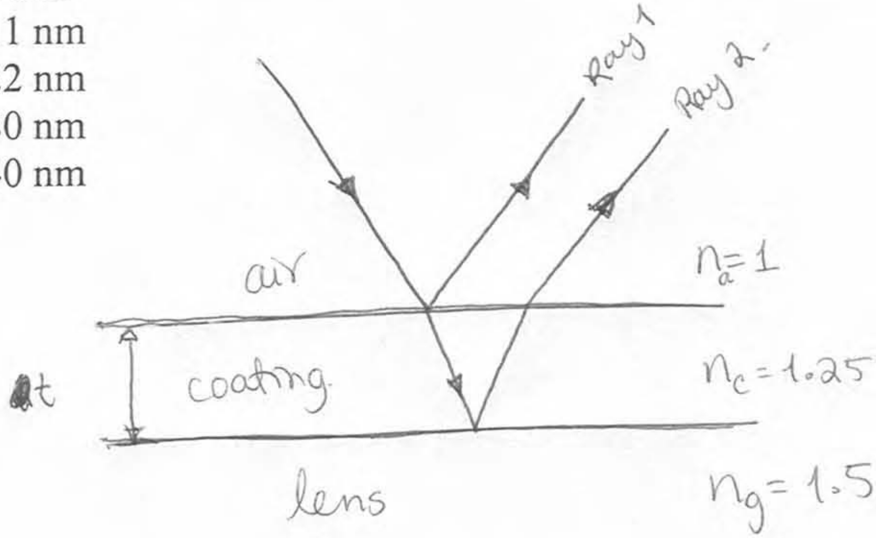
$\mathrm{k} \Rightarrow 10^{3}$ $\mathrm{M} \Rightarrow 10^{6}$ $\mathrm{G} \Rightarrow 10^{9}$ $\mathrm{T} \Rightarrow 10^{12}$ $\mathrm{P} \Rightarrow 10^{15}$

For $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. You apply a material with $n = 1.25$ to a lens ($n_g = 1.5$) to make a nonreflective coating due to destructive interference at a wavelength (in a vacuum) of 555 nm. What is the minimum thickness of the coating that you need?

A) 56 nm
→ B) 111 nm
C) 222 nm
D) 280 nm
E) 140 nm



Since we get $\frac{\pi}{2}$ shift on both reflections, the only overall difference between: the two rays is the extra path covered by ray 2. Since we want destructive interference we want this extra path to be equal to $(m+\frac{1}{2})\lambda_{coating}$

$$2t = (m+\tfrac{1}{2})\lambda_{coating} \longrightarrow \lambda_{coating} = \frac{\lambda_{vacuum}}{n_c}$$

the minimum happens at $m=0$

$$t = \frac{1}{4}\lambda_{coating} = \frac{1}{4}\frac{\lambda_{vacuum}}{n_c} = \frac{1}{4}\frac{555\,nm}{1.25} = 111\,nm$$

2. A hollow, charged spherical conducting thin shell has a radius b and a charge q. Find the potential $V(r)$ everywhere, assuming that $V=0$ at $r=\infty$

A) $V=0,\ r>0$

B) $V=\frac{kq}{r},\ r\geq b;\quad V=0,\ 0<r\leq b$

C) $V=0,\ r\geq b;\quad V=\frac{kq}{r},\ 0<r\leq b$

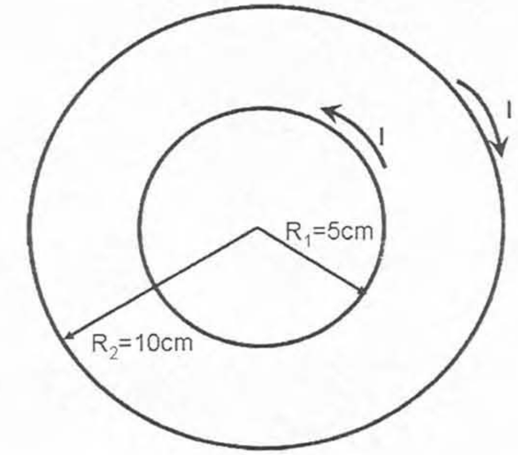
D) $V=\frac{kq}{b},\ r>0$

→ E) $V=\frac{kq}{r},\ r\geq b;\quad V=\frac{kq}{b},\ 0<r\leq b$

When $r \geq b$ the potential behave as the potential for a point charge. Just imagine a Gaussian spherical shell (surface) and the Electric field would be the one for a point charge too, gives you $V=\frac{kq}{r}$ for $r\geq b$:

Follow this same principle, inside the shell at $0<r\leq b$ the electric field drops to zero. But since the potential is 0 at $r=\infty$, the overall accumulated potential inside the shell should be constant and should be equal to $\boxed{V=\frac{kq}{b}}$ the potential at the surface

3. Two concentric loops of radii R_1=5cm and R_2=10cm carry equal anti-parallel currents I=5A as in the figure below. Calculate the magnitude and the direction of the total magnetic field in the center:



→ **A)** 3.14×10^{-5} T out of the page
B) 3.14×10^{-5} T into the page
C) 1.17×10^{-4} T out of the page
D) 1.17×10^{-6} T into the page
E) 6.42×10^{-6} T into the page

The magnetic field at the center of a loop of current is equal to: $B = \frac{\mu_0 I}{2R}$

For $R_1 = 5$ cm we have

$$B_1 = \frac{\mu_0 I}{2R_1} = \frac{(4\pi \times 10^{-7})(5A)}{2(0.05m)} = 6.283 \times 10^{-5} \text{ out of the page}$$

For $R_2 = 10$ cm we

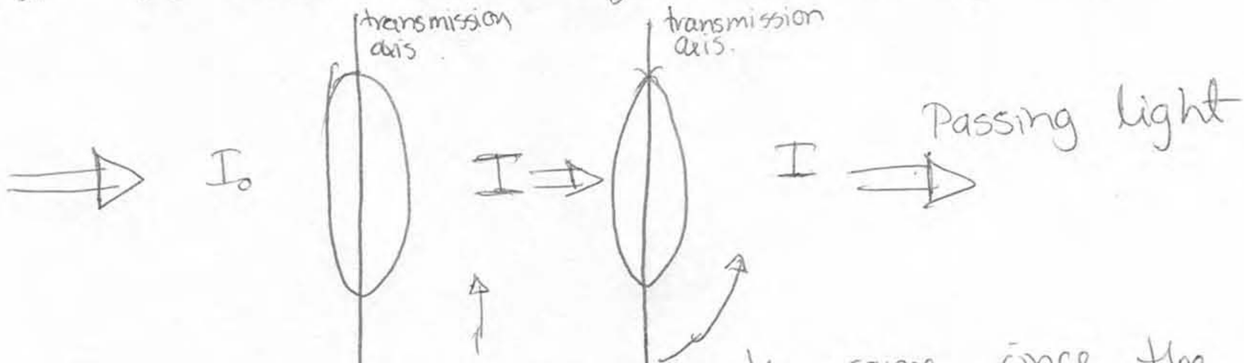
$$B_2 = \frac{\mu_0 I}{2R_2} = \frac{(4\pi \times 10^{-7})(5A)}{2(0.1m)} = -3.14 \times 10^{-5} \text{ into the page}$$

Since they are vectors they substract and we end up with $B_{total} = 3.14 \times 10^{-5}$ out of the page

4. Two ideal polarizing sheets with their planes of polarization parallel to each other pass light of intensity I. If the planes are rotated in such a way that their planes of polarization make an angle of 40.0°, the intensity is approximately

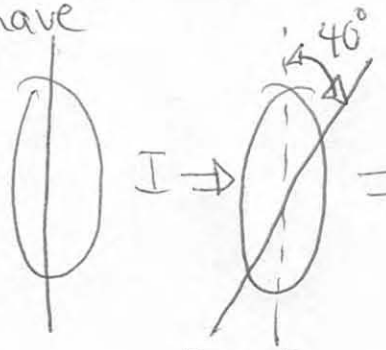
A) $0.293I$
B) $0.383I$
C) $0.502I$
→**D)** $0.585I$
E) $0.770I$

First we have two aligned polarizing sheets



Intensities are the same since the polarizing.

If we rotate one of the polarizers so that their axis of transmisson makes a 40° angle then we have



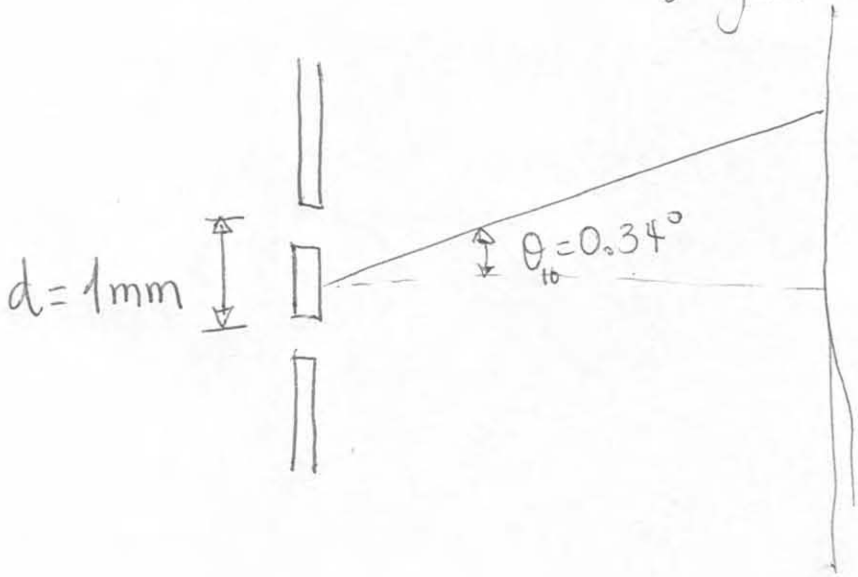
$I_{final} = I \cos^2 40°$

$I_{final} = 0.586 I$

5. Two slits separated by 1.0 mm are illuminated with light of a single unknown wavelength. The **<u>tenth bright</u>** line from the central point of the interference pattern is observed to be at an angle of 0.34°. What is the wavelength of the light?

A) 620 nm
→**B)** 590 nm
C) 560 nm
D) 450 nm
E) 600 nm

Bright fringes:



$$d \sin\theta_{10} = 10\lambda$$

$$\lambda = \frac{d \sin\theta_{10}}{10}$$

$$\lambda = \frac{(1\times10^{-3}\,m)(\sin 0.34^\circ)}{10}$$

$$\boxed{\lambda = 590\,nm}$$

6. An erect object is $2f$ in front of a converging lens of focal length f. The image is:

A) real, inverted, magnified
B) real, erect, same size
→ **C)** real, inverted, same size
D) virtual, inverted, reduced
E) real, inverted, reduced

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad s = 2f$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{f} - \frac{1}{2f} = \frac{2-1}{2f} = \frac{1}{2f}$$

$$\boxed{s' = 2f}$$

positive means <u>real</u>

$$m = -\frac{s'}{s} = -\frac{2f}{2f} = -1$$

– ← <u>inverted</u>

1 ← <u>same size</u>

7. An LR circuit has a resistance $R = 25\ \Omega$, an inductance $L = 5.4$ mH, and a battery of emf = 9.0 V. How much energy is stored in the inductance of this circuit when a steady current is achieved?

A) zero
B) 0.35 J
→**C)** 0.35 mJ
D) 0.70 mJ
E) 0.97 mJ

$$I = \frac{V}{R} = \frac{9.0\,V}{25\,\Omega} = 0.36\,A$$

$$u_L = \frac{1}{2} L I^2 = \frac{1}{2}(5.4\times10^{-3}\,H)(0.36\,A)^2$$

$$= 0.35\times10^{-3}\,J$$

8. You connect a 100-Ω resistor, a 800-mH inductor, and a 10.0-μF capacitor in series across a 60.0-Hz, 120-V (peak) source. Determine the impedance and the approximate resonant frequency of your circuit:

A) 100 Ω and 354 Hz
B) 106 Ω and 60Hz
→ **C)** 106 Ω and 56 Hz
D) 100 Ω and 60 Hz
E) None of these is correct.

$$\omega = 2\pi f$$
$$= 2\pi (60 \text{ Hz})$$

Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$Z = \sqrt{10000 + (301.59 - 265.25)^2} = \underline{106.39\,\Omega}$$

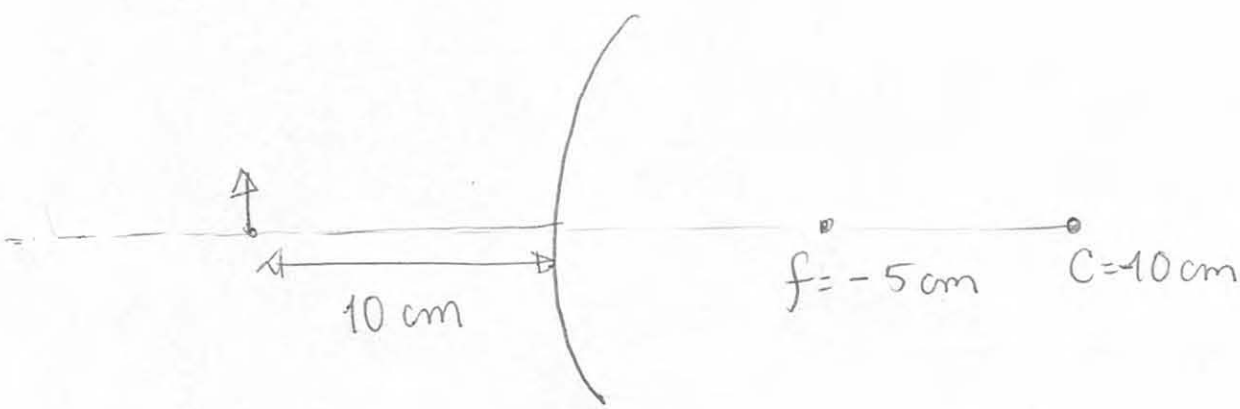
Resonant frequency is

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(800\times10^{-3}\text{H})(10\times10^{-6}\text{F})}}$$

$$f = \frac{\omega}{2\pi} = \underline{56 \text{ Hz}}$$

9. An objet 2 cm high is 10 cm from a convex mirror with a radius of curvature of 10 cm. Locate the position of the image (s') and the height of the image (h')

A) s'= -3.33 cm , h' = 2 cm
→**B)** s'= -3.33 cm , h' = 0.667 cm
C) s'= 3.33 cm , h' = -0.667 cm
D) s'= 3.33 cm , h' = -2 cm
E) s'= -10 cm , h' = 2 cm



$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-5} - \frac{1}{10} = \frac{-2-1}{10} = -\frac{3}{10}$$

$$\boxed{s' = -3.333\ \text{cm}}$$

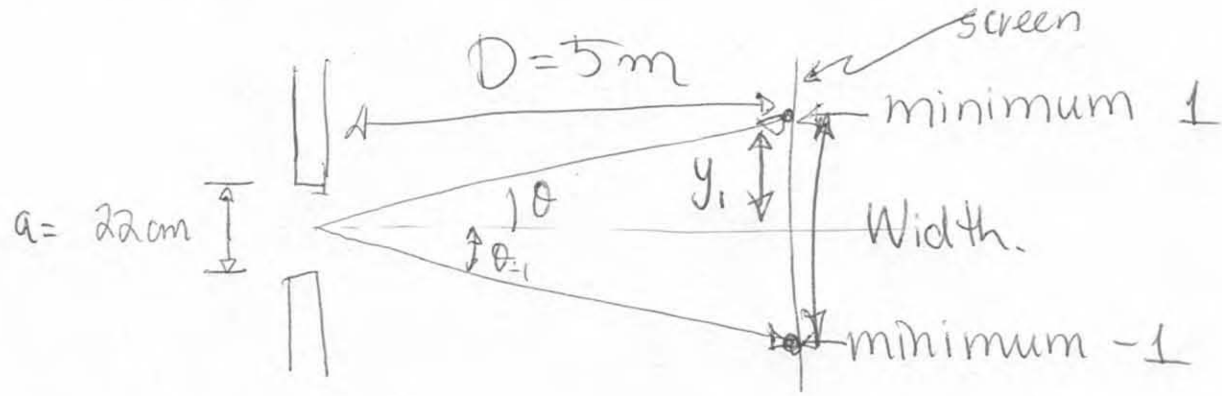
$$m = -\frac{s'}{s} = \frac{h'}{h}$$

$$m = -\frac{-3.333\ \text{cm}}{10\ \text{cm}} = +0.3333$$

$$h' = m\,h = +0.666\ (2\ \text{cm})$$

10. Light of wavelength 450 nm is incident on a narrow slit. The diffraction pattern is observed on a screen 5.0 m from the slit, and the central maximum is observed to have a width of 22 cm. What is the width of the slit?

A) 4.5 μm
B) 5.0 μm
C) 10 μm
→ **D)** 20 μm
E) 0.20 μm



For a single slit we know how to locate the minima only. If we divide the central maximum width by two, then we can figure the y_1 for the first minimum.

$$y_1 = \frac{\text{Width}}{2} = \frac{22\text{cm}}{2} = 11 \text{ cm} = 0.11 \text{ m}.$$

$a \sin\theta_m = m\lambda$ ← minima of a single slit.

$$a = \frac{m\lambda}{\sin\theta_m}$$

$$\tan\theta_1 = \frac{y_1}{D} = \frac{0.11\text{m}}{5\text{m}} = 0.022$$

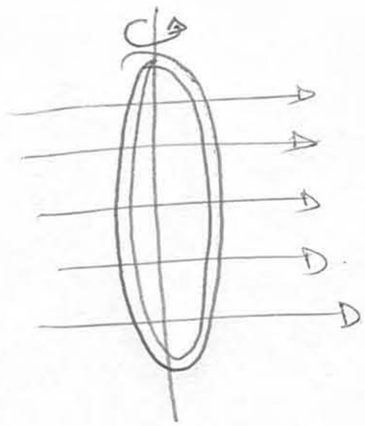
$$\theta_1 = 1.26°$$

$$a = \frac{(1)(450 \text{ nm})}{\sin(1.26°)}$$

$a = 20\ \mu\text{m}$

11. A single loop of wire with a radius of 7.5 cm rotates about a diameter perpendicular to a uniform magnetic field of 1.6 T. To produce a maximum emf of 1.0 V, it should rotate at:

A) 0.0 rad/s
B) 2.7 rad/s
C) 5.6 rad/s
→ **D)** 35 rad/s
E) 71 rad/s



$B = 1.6\ T$

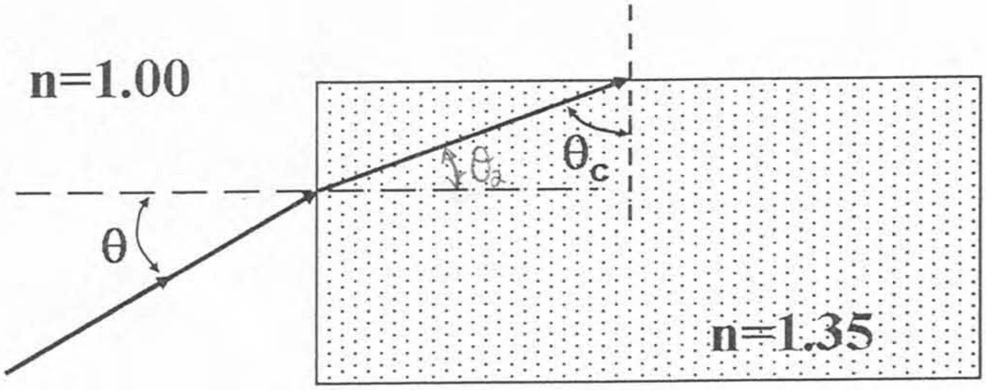
$\phi = AB\cos \omega t$

$\mathcal{E} = -\frac{d\phi}{dt} = +AB(\sin\omega t)\omega$

$\mathcal{E}_{max} = AB\omega = 1.0V$

$\omega = \frac{1V}{\pi r^2 B} = \frac{1V}{\pi (0.075m)^2 (1.6T)} = 35.3\ rad/s$

12. A ray of light traveling in air enters the end of a rectangular block of a material that has an index of refraction $n = 1.35$. The largest value of the angle θ for which total internal reflection occurs at the **upper surface** of the material is approximately



A) 75°
→ **B)** 65°
C) 56°
D) 78°
E) None of these is correct.

For the upper interface we have $n_2 = 1$, $n_1 = 1.35$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1.0}{1.35}\right) = 47.79°$$

For the left hand side interface we have $n_2 = 1.35$, $n_1 = 1$ and $\theta_2 = 90° - \theta_c = 42.2°$ and $\theta = \theta_1$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta = \frac{n_2 \sin\theta_2}{n_1}$$

$$\theta = \sin^{-1}\left(\frac{1.35 \sin(42.2°)}{1}\right)$$

$$\boxed{\theta = 65°}$$

13. A characteristic number for the rate per unit area at which solar energy is delivered to a spot on the Earth is 1000 W/m^2. Use this number to estimate the amplitude of the electric and magnetic fields in the waves that deliver this energy

A) $E_0 = 0.6 \times 10^3$ V/m $B_0 = 0.2 \times 10^{-5}$ T
→ B) $E_0 = 0.9 \times 10^3$ V/m $B_0 = 0.3 \times 10^{-5}$ T
C) $E_0 = 0.2 \times 10^3$ V/m $B_0 = 0.9 \times 10^{-5}$ T
D) $E_0 = 0.3 \times 10^3$ V/m $B_0 = 0.6 \times 10^{-5}$ T
E) $E_0 = 0.6 \times 10^3$ V/m $B_0 = 0.9 \times 10^{-5}$ T

$I = 1000\ W/m^2$

Energy density

$I = u c$

we know that

$u = \varepsilon_0 E_{RMS}^2 = \frac{\varepsilon_0 E_0^2}{2}$

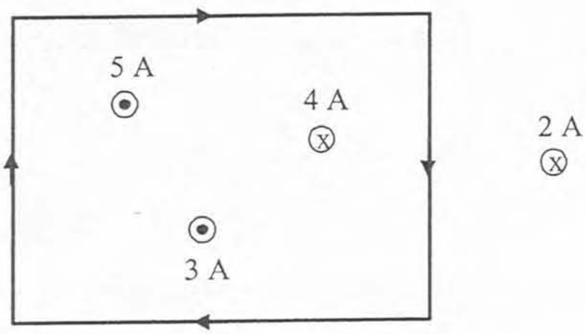
$I = \frac{\varepsilon_0 E_0^2}{2} c$

$\sqrt{\frac{2I}{\varepsilon_0 c}} = E_0 = 867.72\ \frac{N}{C}$

$= 0.9 \times 10^3\ \frac{V}{m}$

$B_0 = \frac{E_0}{c} = 0.3 \times 10^{-5}\ T$

14. What is the value of $\oint_C \vec{B} \bullet d\vec{\ell}$ for the closed curve shown below? (Note that ⊙ represents a current coming out of the page, and ⊗ represents a current going into the page; the arrows indicate the direction of integration).



→ **A)** -5.03×10^{-6} T·m
B) 5.03×10^{-6} T·m
C) -1.51×10^{-6} T·m
D) 1.51 T·m
E) 0 T·m

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{\text{enclosed}}$$

Following the direction of integration then all currents into the page are positive and all currents out of the page are negative. (inside the loop only)

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (+4A - 5A - 3A)$$

$$\oint \vec{B} \cdot \vec{dl} = -5.03 \times 10^{-6} \text{ T·m}$$

15. A 0.120-μF capacitor, initially uncharged, is connected in series with a 10.0-kΩ resistor and a 12.0-V battery of negligible internal resistance. The charge on the capacitor after a very long time is approximately

A) 28.8 μC
B) 14.4 μC
C) 144 μC
D) 2.88 μC
→ **E)** 1.44 μC

$Q = CV$

$Q = (0.120 \times 10^{-6} F)(12V)$

$Q = 1.44\ \mu C$

16. Two small charged objects repel each other with a force F when separated by a distance d. If the charge on each object is reduced to one-fourth of its original value and the distance between them is reduced to $d/2$ the force becomes:

A) $F/16$
B) $F/8$
→ * **C)** $F/4$
D) $F/2$
E) F

q_1 and q_2

separated by a distance d

$$F = K\frac{q_1 q_2}{d^2}$$

Now we have:

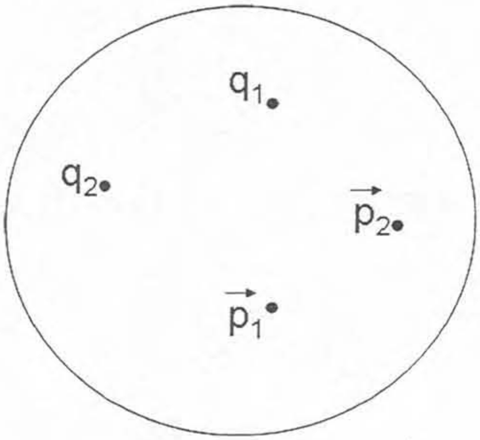
$$q_1' = \frac{q_1}{4} \qquad q_2' = \frac{q_2}{4} \qquad d' = \frac{d}{2}$$

so the force becomes

$$F' = K\frac{q_1' q_2'}{(d')^2} = \frac{K \frac{q_1}{4}\frac{q_2}{4}}{\frac{d^2}{4}} = \frac{1}{4}\left(K\frac{q_1 q_2}{d^2}\right)$$

$$\boxed{F' = \frac{1}{4}F}$$

17. Consider a spherical Gaussian surface of radius 1 m which surrounds two electric dipoles and two charges (one positive and one negative) as shown below. Here q_1=7 nC, q_2= - 4 nC and $p_1=p_2=10^{-10}\ C\cdot m$. What is the net flux through the Gaussian surface?



A) 561 Nm^2/C
B) 7959 Nm^2/C
C) 4649 Nm^2/C
→ **D)** 339 Nm^2/C
E) 2325 Nm^2/C

$$\text{Flux} = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{inside}}{\varepsilon_0}$$

$$Q_{inside} = q_2 + q_1 = 7\times10^{-9}C - 4\times10^{-9}C$$

$$= 3\times10^{-9}C$$

$$\text{Flux} = \frac{(3\times10^{-9}C)}{\varepsilon_0} = 339\ N\cdot m^2/C$$

18. A proton of mass $m=1.67\times10^{-27}$ kg and charge $q=e=+1.6\times10^{-19}$ C moves in a circle of radius r =21cm perpendicular to a magnetic field B= 0.4 T. Find the speed of the proton.

A) 3.0×10^{8} m/s
B) 8.76×10^{-6} m/s
C) 2.19×10^{-6} m/s
D) 2.01×10^{6} m/s
→ **E)** 8.05×10^{6} m/s

$F_{centripetal} = \frac{mv^2}{r}$ $F_{magnetic} = qvB$

$F_{centripeta}$ is provided by $F_{magnetic}$

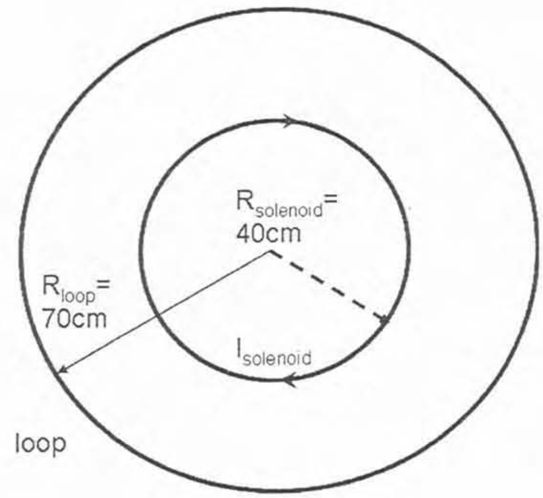
$F_{centri} = F_{magnetic}$

$\frac{mv^2}{r} = qvB$

$v = \frac{qBr}{m} = 8.05\times10^{6}$ m/s

19. A long solenoid with a radius of 40 cm has 10000 turn/m.
a) Find the value of the B field generated inside the solenoid (on the solenoid's axis) when a current of I=10 A flows clockwise (seen from the end of the solenoid).

b) A single circular loop of radius 70 cm is placed around the solenoid, the central axis of the loop and the solenoid coinciding. If the magnetic field in the solenoid decreases uniformely from the value found in part a) to 0.025 T in 0.02 s. Determine the direction of the induced current in the loop.



A) a) B= 0.13 T b) counter-clockwise
B) a) B= 0.13 T b) clockwise
C) a) B= 0.048 T b) counter-clockwise
D) a) B= 0.048 T b) clockwise
E) a) B= 0.084 T b) counter-clockwise

a) $B = \mu_0 n I = \underline{0.13 T}$

b) Since the B field is going in to the page and it's decreasing. According to lenz's law the induced current should oppose the loss.

The induced current has to flow <u>clockwise</u> to oppose the change

20. A uniform electric field of 300 N/C makes an angle of 25° with the dipole moment of an electric dipole. If the moment has a magnitude of $2\times10^{-9}\,\mathrm{C\cdot m}$, the torque exerted by the field has a magnitude of:

A) $6.7\times10^{-12}\,\mathrm{N\cdot m}$
→ **B)** $2.5\times10^{-7}\,\mathrm{N\cdot m}$
C) $5.4\times10^{-7}\,\mathrm{N\cdot m}$
D) $6.0\times10^{-7}\,\mathrm{N\cdot m}$
E) $2.8\times10^{-12}\,\mathrm{N\cdot m}$

$\tau = \vec{p}\times\vec{E} = pE\sin\theta = 2.5\times10^{-7}\ \mathrm{N\cdot m}$