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Nuclear Reactions in Micro/Nano-Scale Metal Particles

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Abstract Low-energy nuclear reactions in micro/nano-scale metal particles are described based on the theory of Bose–Einstein condensation nuclear fusion (BECNF). The BECNF theory is based on a single basic assumption capable of explaining the observed LENR phenomena; deuterons in metals undergo Bose–Einstein condensation. The BECNF theory is also a quantitative predictive physical theory. Experimental tests of the basic assumption and theoretical predictions are proposed. Potential application to energy generation by ignition at low temperatures is described. Generalized theory of BECNF is used to carry out theoretical analyses of recently reported experimental results for hydrogen–nickel system.

1 Introduction

Over the last two decades, there have been many publications reporting experimental observations of excess heat generation and anomalous nuclear reactions occurring in metals at ultra-low energies, now known as *low-energy nuclear reactions* (LENR). Theoretical explanations of the LENR phenomena will be described based on the theory of Bose-Einstein condensation nuclear fusion (BECNF) in micro/nano-scale metal particles [1–3]. The BECNF theory is based on a single basic assumption capable of explaining the observed LENR phenomena; deuterons in metals undergo Bose–Einstein condensation. While the BECNF theory is able to make general qualitative predictions concerning LENR phenomena it is also a quantitative predictive physical theory. Proposed experimental tests of the basic assumption and theoretical predictions will be described. Although the BECNF theory indicates possibilities of scaling up heat generation under optimal conditions, experimental tests of theoretical predictions are needed for confirmation, improvement of the BECNF theory and to clarify potential practical applications. The BECNF theory was generalized for the case of two species of Bosons [4]. Generalized theory of BECNF is used to carry out theoretical analyses of recently reported experimental results for hydrogen–nickel system.

2 Anomalous Experimental Results

The conventional deuterium fusion in free space proceeds via the following nuclear reactions:

$\{1\}\ D + D \rightarrow p\,(3.02\,\mathrm{MeV}) + T\,(1.01\,\mathrm{MeV})$,
$\{2\}\ D + D \rightarrow n\,(2.45\,\mathrm{MeV}) + {}^{3}\mathrm{He}\,(0.82\,\mathrm{MeV})$,
$\{3\}\ D + D \rightarrow {}^{4}\mathrm{He} + \gamma\,(23.8\,\mathrm{MeV})$.

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The cross-sections for reactions {1}–{3} are expected to be extremely small at low energies ($\leq$10) eV due to the Gamow factor arising from Coulomb barrier between two deuterons. The measured cross-sections have branching ratios: $(\sigma\{1\}, \sigma\{2\}, \sigma\{3\}) \approx (0.5, 0.5, \sim 10^{-6})$.

From many experimental measurements by Fleischmann and Pons [5] and many others over 20 years since then (see references in Refs. [1–3,6]), the following experimental results have emerged. At ambient temperatures or low energies($\leq$10) eV, deuterium fusion in metal proceeds dominantly via the following reactions:

{4} $D(m) + D(m) \rightarrow {}^{4}\mathrm{He}(m) + 23.8\,\mathrm{MeV}(m)$,

where m represents a host metal lattice or metal particle.

3 Deuteron Mobility in Metal

Development of Bose–Einstein condensate theory of deuteron fusion in metal is based upon a single hypothesis that deuterons in metal are mobile and hence are capable of forming Bose–Einstein condensates.

Experimental proof of proton (deuteron) mobility in metals was first demonstrated by Coehn in his hydrogen electro-migration experiment [7,8]. The significance of Coehn's experimental results [7] is emphasized by Bartolomeo et al. [9]. A theoretical explanation of Coehn's results [7] is given by Isenberg [10]. The Coehn's experimental fact is not well known in review articles and textbooks.

There are other experimental evidences [11–15] that heating and/or applying an electric field in a metal causes protons and deuterons in a metal to become mobile, thus leading to a higher density for quasi-free mobile deuterons in a metal. It is expected that the number of mobile deuterium ions will increase, as the loading ratio D/metal of deuterium atoms increases and becomes larger than one, D/metal $\geq$1.

Mobility of deuterons in a metal is a complex phenomenon and may involve a number of different processes [15]: coherent tunneling, incoherent hopping, phonon-assisted processes, thermally activated tunneling, and over-barrier jump/fluid like motion at higher temperatures. Furthermore, applied electric fields as in electrolysis experiments can enhance the mobility of absorbed deuterons.

4 BEC Fraction of Deuterons in Metal

Fraction of deuterons in a metal satisfying BEC condition can be estimated as a function of the temperature. The BEC condensate fraction $F(T) = N_{\mathrm{BEC}}/N$ can be calculated from integrals,

$$N_{\mathrm{BEC}} = \int_{0}^{E_c} n(E)N(E)dE \text{ and } N = \int_{0}^{\infty} n(E)N(E)dE, \tag{1}$$

where $n(E)$ is either Bose–Einstein or Maxwell–Boltzmann distribution function, $N(E)$ is the density of (quantum) states, and E_C is the critical kinetic energy of deuteron satisfying the BEC condition $\lambda_c = d$, where λ_c is the de Broglie wavelength of deuteron corresponding to E_C and d is the average distance between two deuterons. For $d = 2.5$Å, we obtain $F(T = 300\,°\mathrm{K}) \approx 0.084$ (8.4 %), $F(T = 77.3\,°\mathrm{K}) \approx 0.44$(44 %), and $F(T = 20.3\,°\mathrm{K}) \approx 0.94$ (94 %). At $T = 300\,°\mathrm{K}$, $F = 0.084$ (8.4 %) is not large enough to form BEC since motions of deuterons are limited to several lattice sites and the probability of their encounters are very small. On the other hand, at liquid nitrogen (77.3 °K) and liquid hydrogen (20.3 °K) temperatures, probability of forming BEC of deuterons is expected to be $\Omega \approx 1$. This suggests that experiments at these low temperatures can provide tests for enhancement of the reaction rate R_t, Eq. (4) below, as predicted by BECNF theory.

5 Bose–Einstein Condensation Theory of Deuteron Fusion in Metal

For applying the concept of the BEC mechanism to deuteron fusion in a micro/nano-scale metal particle, we consider N identical charged Bose nuclei (deuterons) confined in an ion trap (or a metal grain or particle). Some fraction of trapped deuterons are assumed to be mobile as discussed above. The trapping potential is 3-dimensional (nearly-sphere) for micro/nano-scale metal particles, or quasi 2-dimensional (nearly hemi-sphere) for micro-scale metal grains, both having surrounding boundary barriers. The barrier heights or potential depths are expected to be an order of energy (≤ 1) eV required for removing a deuteron from a metal grain or particle. For simplicity, we assume an isotropic harmonic potential for the ion trap to obtain order of magnitude estimates of fusion reaction rates.

N-body Schrodinger equation for the system is given by

$$H\Psi = E\Psi \tag{2}$$

with the Hamiltonian H for the system given by

$$H = \frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_i + \frac{1}{2}m\omega^2\sum_{i=1}^{N} r_i^2 + \sum_{i<j}\frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|}, \tag{3}$$

where m is the rest mass of the nucleus. Only two-body interactions (Coulomb and nuclear forces) are considered since we expect that three-body interactions are expected to be much weaker than the two-body interactions.

The approximate ground-state solution of Eq. (2) with H given by Eq. (3) is obtained using the equivalent linear two-body method [16,17]. The use of an alternative method based on the mean-field theory for bosons yields the same result (see Appendix in Ref. [18]). Based on the optical theorem formulation of low energy nuclear reactions [19], the ground-state solution is used to derive the approximate theoretical formula for the deuteron–deuteron fusion rate in an ion trap (micro/nano-scale metal grain or particle). The detailed derivations are given elsewhere [18,20].

Our final theoretical formula for the total fusion rate R_t for large N case is given by [18,20]

$$R_t = 4(3/4\pi)^{3/2}\Omega A N_D \frac{N}{D_{\text{trap}}^3}, \tag{4}$$

where N is the average number of Bose nuclei (deuterons) in a trap, D_{trap} is the average diameter of the trap, $A = 2Sr_B/(\pi\hbar)$, $r_B = \hbar^2/(2\mu e^2)$, and S is the S-factor for the nuclear fusion reaction between two deuterons. N_D is the total number of deuterons and $N_{\text{trap}} = N_D/N$ is the total number of traps. For $D(d,p)T$ and $D(d,n)^3He$ reactions, we have $S \approx 55$ keV-barn. We expect also $S \approx 55$ keV-barn or larger for reaction {4}. $A = 0.77 \times 10^{-16}\text{cm}^3/\text{s}$ for S = 55 keV-barn. Only two unknown parameters are (i) the probability of the BEC ground state occupation, Ω, and (ii) the S-factor. Equation (4) shows that the total fusion rates, R_t, are maximized when $\Omega = 1$.

Equation (4) provides an important result that nuclear fusion rates R_t for large N case do not depend on the Gamow factor in contrast to the conventional theory for two-body nuclear fusion in free space. This could provide explanations for overcoming the Coulomb barrier and for the observed anomalous effects for low-energy nuclear reactions in metals. This is consistent with the conjecture noted by Dirac [21] and used by Bogolubov [22] that boson creation and annihilation operators can be treated simply as numbers when the ground state occupation number is large. This implies that for large N each charged boson behaves as an independent particle in a common average background potential and the Coulomb interaction between two charged bosons is suppressed. This provides theoretical explanations for the observation {4} as well as other observed experimental results.

For the large N case, the deuteron-deuteron reaction {4} in a BEC state proceeds via

$$\Psi_{\text{BEC}}[(D + D) + (N-2)D's] \rightarrow \Psi^*_{\text{BEC}}[^4\text{He} + (N-2)D's]\ (Q = 23.84\,\text{MeV}), \tag{5}$$

where the Q value of 23.84 MeV is shared by ^{4}He and all $D's$ in the BEC state, thus maintaining the momentum conservation in the final state. This is a BEC energy dissipation (BECED) mechanism. This implies that the deuteron BEC state undergoes a micro/nano-scale explosion (*nano-explosion* or *micro-explosion*). For a micro/nano-scale metal particle of 10 nm diameter containing ~3.6 $\times 10^4$ deuterons, each deuteron or ^{4}He will gain only ~6.5 keV kinetic energy, if the excess kinetic energy of 23.84 MeV is shared equally. For a larger metal particle, ~6.5 keV is further reduced. This BECED mechanism can provide an explanation for constraints imposed by the secondary neutron, gamma-rays, and X-rays, produced by energetic ^{4}He, as described by Hagelstein [23].

Other exit channels, {1} and {2}, are expected to have much less probabilities than that of the exit channel {4} (described by Eq. (5)), since both {1} and {2} involve centrifugal and Coulomb barrier transmissions of exit particles in the exit channels, while {4} (described by Eq. (5)) does not.

6 Proposed Experimental Tests of the BECNF Theory

Since the BECNF theory is a quantitative predictive physical theory, it is important to test its hypothesis and predictions for confirmation, improvement of the BECNF theory and to clarify potential practical applications. BECNF theory is based on one single physical hypothesis that mobile deuterons in a metal grain/particle form a Bose–Einstein condensate. Therefore, it is important to explore experimental tests of this basic hypothesis. Two types of experimental tests (Experiments 1 and 2) are proposed as described below. For both types of experiments, the dependence on the temperature and pressure are to be measured. A third experiment is proposed (Experiment 3) for exploring a practical application of the BECNF theory.

Proposed Experiment 1 As is the case for the atomic BEC experiments, experiments are proposed to measure the velocity distribution of deuterons in metal using low-energy neutron scattering. The measurements can be carried out using low-energy neutrons available from nuclear reactors or from spallation neutron sources. An enhancement of low-velocity deuterons in the deuteron velocity distribution is expected when the BEC of deuterons occurs. This experimental demonstration of the BEC of deuterons in a metal may lead to a new discovery. In 1995, this type of experiments was used to establish the existence of the BEC of atoms in a magnetic trap at extremely low temperatures, for which the Nobel prize was awarded in 2000.

Proposed Experiment 2 To explore the superfluidity of the BEC of deuterons in metal, experiments are proposed to measure the diffusion rates of deuterons and protons in a metal as a function of temperature. When the BEC of deuterons in a metal occurs, it is expected that the deuteron diffusion rate will increase substantially more than that of proton. We will explore a number of other experimental methods for observing the superfluidity. Experimental demonstration of the superfluidity of deuterons in the BEC state in metal may lead to a new discovery. In 1996, the Nobel prize was awarded for the discovery of superfluidity of helium-3.

Proposed Experiment 3 To explore possibilities of constructing a practical BECNF reactor for energy generation, both experimental and theoretical investigations are proposed to study the possibility of BECNF mini-explosion (or ignition) at extremely low temperatures. At ^{4}He liquid temperature, from estimates of reaction rates using Eq. (4), DD fusions are expected to occur nearly simultaneously from each of micro/nano-scale metal particles contained in a bulk volume. This can cause a mini-explosion (or ignition). An ignition fuel of $\sim 1\,\mathrm{cm}^3$ volume containing $\sim 10^{18}$ of ∼10 nm metal particles (each loaded with $\sim 10^{4-5}$ deuterons) could be used to ignite $\sim 10^{18}$ DD fusions at ^{4}He liquid temperature in a very short time period. If the proposed experimental test is proved to be successful, the ignition fuel can be used in a reaction chamber similar to the ignition chamber containing a cryogenic-target at the National Ignition Facility, Livermore National Laboratory [24].

7 Application to Hydrogen–Nickel Reactions

As an application of the BECNF theory, a generalized theory of BECNF can be used to carry out theoretical analyses of experimental results for hydrogen–nickel system. The BECNF theory was generalized for the case of two species of Bosons [4].

Since 1993, observations of hydrogen–nickel reactions in hydrogen-gas loading experiments were reported by Focardi, Piantelli, and others [25,26], who used metal plates and metal cylinders of Ni and Ni-alloys. In a particular experiment, Focardi et al. report that excess output power of ∼40 W was produced for 278 days with input power of ∼100 W at sample temperatures of ∼450 °C [25].

Recently, there were two positive demonstrations (January and March, 2011) of a heat generating device called *Energy Catalyzer* [27] in which micro/nano-scale nickel particles/powders were used. The Energy Catalyzer is an apparatus built by inventor Andrea Rossi, Italy. The patent application [27] states that the device transforms energy stored in its fuel (hydrogen and nickel) into heat by means of nuclear reaction of the two fuel components, with a consequent observed production of copper [27,28]. According to Rossi's patent application [27], heating of the sample is accomplished by an electric resistance heater. Details of March 2011 demonstration were reported by Essen and Kullander [29]. They report that with electric input power of ∼0.3 kW, the device produced output power of ∼4.69 kW (∼14× the input power) during a period of ∼5 h and 45 min [29]. The device was operated at temperatures of ∼400°C with hydrogen pressure of ∼25 bars. The report [29] also contains references to January 2011 demonstration.

The generalized BECNF theory [4] can be applied to the case of hydrogen–nickel fusion reactions observed in Rossi's device (the energy catalyzer) [27] under the following two conditions: (1) Ni alloys and/or Ni metal/alloy oxides are formed in the surface regions of micro/nano-scale Ni particles, so that Ni atoms/nuclei become mobile with a sufficiently large diffusion coefficient and (2) local magnetic field is very weak in the surface regions, providing a suitable environment in which two neighboring protons can couple their spins anti-parallel to form spin-zero singlet state (S = 0). Relatively low Curie temperature [nickel has the Curie temperature of 631°K (∼358 °C)] is expected to help to maintain the weak magnetic field in the surface regions. If Rossi's device is operated at temperatures greater than the Curie temperature ∼358 °C and with hydrogen pressures of up to ∼25 bars, the conditions (1) and (2) may have been achieved in Rossi's device.

The mobility of Ni atoms/nuclei (condition (1)) is enhanced by the use of an electric resistance heater to maintain higher temperatures. This may provide a suitable environment in which more of both Ni atoms/nuclei and protons become mobile, thus creating a favorable environment for maintaining two species of Bosons (Ni nuclei and composite Bosons of paired two protons). If the velocities of mobile Ni atoms/nuclei under the condition (1) are sufficiently slow, their de-Broglie wavelengths become sufficiently large and may overlap with neighboring two-proton composite Bosons which are also mobile, thus creating Bose–Einstein condensation of two species of Bosons. The generalized BECNF theory can now be applied to these two-species of Bosons and provides a mechanism for the suppression/cancellation of the Coulomb barrier, as shown in Ref. [4].

Once the Coulomb barrier is overcome in the entrance reaction channel, many possible allowed exit reaction channels may become open such as reactions (i) $^{A}\mathrm{Ni}(2p(S=0),\,p)^{A+1}\mathrm{Cu}$, with even A = 58, 60, 62 and 64. There are other exit reaction channels which are radiation-less, such as reactions (ii) $^{A}\mathrm{Ni}(2p(S=0),\,\alpha)^{A-2}\mathrm{Ni}$, (even A = 58, 60, 62, and 64) [30]. The effect of the secondary nuclear reactions by emitted charged particles with large Q values are expected to be substantially reduced due to the BECED mechanism described at the end of Sect. 5. This will reduce reaction rates for producing neutron, gamma-rays, and X-rays from the secondary reactions by energetic charged particles. A more detailed description of theoretical analysis and predictions is given in a recent paper [31].

In order to explore validity and to test predictions of the generalized BECNF theory for the hydrogen–metal system, it is very important to carry out further experiments in order to establish what are exact inputs and outputs of each experiment. If the entrance and exit reaction channels are established experimentally, we can investigate selection rules as well as estimates of the reaction rates for different exit reaction channels, based on the generalized BECNF theory [1–4]. Once these experimental results are established, further application of the generalized BECNF theory can be made for the purpose of confirming the theoretical mechanism and making theoretical predictions, which can then be tested experimentally.

8 Conclusions

The BECNF theory is based on a single physical assumption of the new basic concept of BEC of deuterons in metals. It provides consistent theoretical explanations for experimental observations. Two experiments are proposed for testing the basic concept of BEC of deuterons in metals. As a potential practical application, experimental test is proposed for a mini-explosion of a bulk collection of micro/nano-scale deuterated metal particles at ^{4}He liquid temperature. If the proposed experimental test is successful, the ignition fuel can be used in a reaction chamber similar to the ignition chamber containing a cryogenic-target at the National Ignition Facility, Livermore National Laboratory [24]. Most recently, generalized BECNF theory is used to make theoretical predictions of BECNF processes for hydrogen–nickel reactions. Theoretical predictions provide suggestions for future experiments which in turn can provide new useful data for refinements of the theory. In view of the impending world energy crisis, the proposed experimental tests of the BECNF processes are urgently needed as LENR phenomena may well represent a viable long-term alternative form of clean energy.

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