NUCLEAR FUSION FOR BOSE NUCLEI CONFINED IN ION TRAPS

NUCLEAR REACTIONS IN SOLIDS

KEYWORDS: *ion trap nuclear fusion, deuteron-deuteron fusion*

YEONG E. KIM* and ALEXANDER L. ZUBAREV
Purdue Nuclear and Many-Body Theory Group
Purdue University, Department of Physics
West Lafayette, Indiana 47907

Received August 18, 1999
Accepted for Publication October 14, 1999

Nuclear fusion of integer spin nuclei confined in an isotropic ion trap is investigated. Solutions of the ground state for charged bosons trapped in the isotropic harmonic oscillator potential are calculated using the equivalent linear two-body method for many-body problems, which is based on an approximate reduction of the many-body Schrödinger equation by the use of a variational principle. Using the ground-state wave function, theoretical estimates of probabilities and rates for nuclear fusion for Bose nuclei confined in ion traps are obtained. Numerical estimates for fusion rates are presented for the case of deuteron-deuteron fusion.

I. INTRODUCTION

During the past few decades, ion trap devices (Paul and Penning traps)[1] have been playing important roles in carrying out some extraordinary experiments in atomic physics, such as those where a single charged particle is confined indefinitely in a small region of space. Some important applications of ion traps have been made to analytical chemistry and ion mass spectroscopy. In the future, we may expect new types of experiments using both Paul and Penning traps involving 10^5 to 10^6 trapped ions or exotic nuclei.[1] For neutral atoms, magnetic traps with new cooling techniques have been used to observe the Bose-Einstein condensation of neutral atoms (Ref. 2 and references therein).

In this paper, we investigate theoretically different aspects of the properties of identical integer-spin nuclei ("Bose" nuclei) confined in ion traps by approximating the ion trap with an isotropic harmonic oscillator potential for simplicity. We report the results of our theoretical investigation on the feasibility of nuclear fusion in such setups ("ion trap nuclear fusion") using the recently developed equivalent linear two-body (ELTB) method for many-body problems.[3] The ELTB method is based on an approximate reduction of the many-body Schrödinger equation by the use of a variational principle.

In Sec. II, we describe the ELTB method for obtaining solutions of the ground state for charged bosons confined in the isotropic trap. In Sec. III, we describe an approximation for obtaining the nuclear reaction cross section for two nuclei, using the optical theorem and the Fermi potential. In Sec. IV, we derive our formulas for nuclear fusion probabilities and for the rates of the ion trap nuclear fusion using the results of Secs. II and III. The formulas obtained in Sec. IV are applied to deuteron-deuteron fusion in ion traps in Sec. V. A summary and conclusions are given in Sec. VI.

II. GROUND-STATE SOLUTION

In this section we consider N identical charged Bose nuclei confined in an ion trap. For simplicity, we assume an isotropic harmonic potential for the ion trap to obtain orders-of-magnitude estimates. The Hamiltonian for the system is then

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_i + \frac{1}{2}m\omega^2\sum_{i=1}^{N} r_i^2 + \sum_{i<j}\frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} , \quad (1)$$

where m is the rest mass of the nucleus.

For ground-state wave function Ψ, we use the following approximation[1]:

$$\Psi(\vec{r}_1, \ldots \vec{r}_N) \approx \tilde{\Psi}(\rho) = \frac{\Phi(\rho)}{\rho^{(3N-1)/2}} , \quad (2)$$

*E-mail: yekim@physics.purdue.edu

where

$$\rho = \left[\sum_{i=1}^{N} r_i^2\right]^{1/2} . \quad (3)$$

In Ref. 3 it has been shown that approximation (2) yields good results for the case of large N. Dirac[4] and Bogolubov[5] noted and used the fact that for large N, the ground-state creation and annihilation operators, a_0 and a_0^+, can be treated simply as a number because their commutator $[a_0, a_0^+] = 1$ is small compared to their value, $a_0 \approx a_0^+ \approx \sqrt{N}$.

By requiring that $\tilde{\Psi}$ must satisfy a variational principle $\delta \int \tilde{\Psi}^* H \tilde{\Psi}\, d\tau = 0$ with a subsidiary condition $\int \tilde{\Psi}^* \tilde{\Psi}\, d\tau = 1$, we obtain the following Schrödinger equation for the ground-state wave function $\Phi(\rho)$:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2} + \frac{m}{2}\omega^2\rho^2 + \frac{\hbar^2}{2m}\frac{(3N-1)(3N-3)}{4\rho^2} + V(\rho)\right]\Phi = E\Phi , \quad (4)$$

where[1]

$$V(\rho) = \frac{2N\Gamma(3N/2)}{3\sqrt{2\pi}\Gamma(3N/2 - 3/2)} \frac{e^2}{\rho} . \quad (5)$$

Instead of the variable ρ in the Schrödinger equation (4), we introduce a new quantity $\tilde{\rho}$, defined as

$$\tilde{\rho} = \sqrt{\frac{m\omega}{\hbar}}\,\rho . \quad (6)$$

Substitution of Eq. (6) into Eq. (4) leads to the following equation:

$$\frac{\hbar\omega}{2}\left[-\frac{d^2}{d\tilde{\rho}^2} + \tilde{\rho}^2 + \frac{(3N-1)(3N-3)}{4\tilde{\rho}^2} + \frac{\tilde{\gamma}}{\tilde{\rho}}\right]\Phi = E\Phi , \quad (7)$$

where

$$\tilde{\gamma} = \alpha\sqrt{\frac{mc^2}{\hbar\omega}}\frac{4N\Gamma(3N/2)}{3\sqrt{2\pi}\Gamma(3N/2 - 3/2)} \quad (8)$$

with $\alpha = e^2/(\hbar c) \approx 1/137$. Now, we seek the ground-state solution of Eq. (7) in the following form:

$$\Phi(\tilde{\rho}) = \sum_i c_i \tilde{\rho}^{\frac{3N-1}{2}} e^{-(\tilde{\rho}/\alpha_i)^2/2} , \quad (9)$$

where c_i are solutions of the following equations:

$$\sum_l H_{il} c_l = E \sum_l \lambda_{il} c_l , \quad (10)$$

with

$$\lambda_{il} = \left[\frac{2\alpha_i\alpha_l}{\alpha_i^2 + \alpha_l^2}\right]^{3N/2} , \quad (11)$$

$$H_{il} = \frac{N\hbar\omega\lambda_{il}}{2}\left[\frac{3(1 + \alpha_i^2\alpha_l^2)}{\alpha_i^2 + \alpha_l^2} + \zeta\left(\frac{\alpha_i^2 + \alpha_l^2}{2\alpha_i^2\alpha_l^2}\right)^{1/2}\right] , \quad (12)$$

and

$$\zeta = \sqrt{\frac{mc^2}{\hbar\omega}}\frac{4\alpha}{3\sqrt{2\pi}}\frac{\Gamma\left(\frac{3N}{2} - \frac{1}{2}\right)}{\Gamma\left(\frac{3N}{2} - \frac{3}{2}\right)} . \quad (13)$$

For the case of large N, λ_{il} reduces to the δ-function

$$\lambda_{il} \approx \delta_{il} , \quad (14)$$

and hence we obtain

$$H_{il} \approx E\delta_{il} . \quad (15)$$

Using Eq. (15), we have for the ground-state energy

$$E = \frac{\hbar\tilde{\omega}N}{2}\left[\frac{3(1 + \alpha_t^4)}{2\alpha_t^2} + \frac{\zeta}{\alpha_t}\right] , \quad (16)$$

where the parameter α_t is a solution of the following equation:

$$\frac{dE}{d\alpha_t} = 0 . \quad (17)$$

For the case of large N, we can neglect the kinetic energy term in Eq. (16) to obtain

$$E = \frac{\hbar\omega N}{2}\left[\frac{3\alpha_t^2}{2} + \frac{\zeta}{\alpha_t}\right] . \quad (18)$$

Substitution of Eq. (18) into Eq. (17) leads to

$$\alpha_t = \left(\frac{\zeta}{3}\right)^{1/3} , \quad (19)$$

and from Eq. (18) we obtain for the ground-state energy

$$E = \frac{\hbar\omega N}{4} 3^{4/3}\zeta^{2/3} , \quad (20)$$

where for the case of large N, ζ reduces to the following expression:

$$\zeta \approx 2\sqrt{\frac{mc^2}{2\pi\hbar\omega}}\,\alpha N . \quad (21)$$

III. IMAGINARY PART OF THE FERMI POTENTIAL

The total elastic nucleus-nucleus amplitude can be written as

$$f(\theta) = f^c(\theta) + \tilde{f}(\theta) \ , \tag{22}$$

where $f^c(\theta)$ is the Coulomb amplitude and $\tilde{f}(\theta)$ can be expanded in partial waves[6]:

$$\tilde{f}(\theta) = \sum_l (2l+1) e^{2i\delta_l^c} f_l^{n(el)} P_l(\cos\theta) \ . \tag{23}$$

In Eq. (23), δ_l^c is the Coulomb phase shift, $f_l^{n(el)} = (S_l^n - 1)/2ik$, and S_l^n is the l'th partial S-matrix for the nuclear part. For low energy, we can write[7]

$$\mathrm{Im} f_l^{n(el)} \approx \frac{k}{4\pi} \sigma_l^r \ , \tag{24}$$

where σ_l^r is the partial wave reaction cross section. For the dominant contribution of only the s-wave, we have

$$\mathrm{Im} f_0^{n(el)} \approx \frac{k}{4\pi} \sigma^r \ , \tag{25}$$

where σ^r is conventionally parameterized as

$$\sigma^r = \frac{S}{E} e^{-2\pi\eta} \ , \tag{26}$$

where

$$\eta = \frac{1}{2kr_B}$$

$$r_B = \frac{\hbar^2}{2\mu e^2}$$

$$\mu = m/2$$

S = S factor for the nuclear fusion reaction between two nuclei.

In terms of the partial-wave t-matrix, the elastic-scattering amplitude $f_l^{n(el)}$ can be written as[7]

$$f_l^{n(el)} = -\frac{2\mu}{\hbar^2 k^2} \langle \psi_l^c | t_l | \psi_l^c \rangle \ , \tag{27}$$

where ψ_l^c is the Coulomb wave function.

Introducing a new quantity U as the imaginary part of t_o,

$$U = \mathrm{Im}(t_0) \ , \tag{28}$$

we have

$$\frac{k}{4\pi} \sigma^r = -\frac{2\mu}{\hbar^2 k^2} \langle \psi_0^c | U | \psi_0^c \rangle \ . \tag{29}$$

For the $l = 0$ case, the Coulomb wave function ψ_0^c is given by

$$\psi_0^c(r) = C_0(\eta) M_{i\eta,1/2}(2ikr)/2i \ , \tag{30}$$

where

$$C_0(\eta)^2 = \frac{2\pi\eta}{\exp(2\pi\eta) - 1} \tag{31}$$

and $M_{i\eta,1/2}(2ikr)$ is the Whittaker function.

Using Eqs. (30) and (31), we can write Eq. (29) approximately as

$$\frac{k}{4\pi} \sigma^r = -\frac{2\mu}{\hbar^2} C_0(\eta)^2 \int r dr \int r' dr' \, U(r,r') \ . \tag{32}$$

The use of Eq. (26) in Eq. (32) leads to

$$\int r dr \int r' dr' U(r,r') = \frac{-S r_B}{4\pi^2} \ . \tag{33}$$

For our case of N Bose nuclei, to account for a short-range nature of nuclear forces between two nuclei, we introduce the following Fermi pseudopotential $V^F(\vec{r})$:

$$\mathrm{Im}\, V^F(\vec{r}) = -\frac{A\hbar}{2} \delta(\vec{r}) \ , \tag{34}$$

where the nuclear rate constant A is given by

$$A = \frac{2Sr_B}{\pi\hbar} \ . \tag{35}$$

IV. FUSION PROBABILITY AND RATES

For N identical Bose nuclei confined in an ion trap, the nucleus-nucleus fusion rate is determined from the trapped ground-state wave function Ψ as

$$\tilde{R} = -\frac{2}{\hbar} \frac{\sum_{i<j} \langle \Psi | \mathrm{Im}\, V_{ij}^F | \Psi \rangle}{\langle \Psi | \Psi \rangle} \ , \tag{36}$$

where $\mathrm{Im}\, V_{ij}^F$ is the imaginary part of the Fermi potential, given by Eq. (34).

The substitution of Eq. (2) into Eq. (36) yields

$$\tilde{R} = \frac{AN(N-1)\Gamma(3N/2)}{2(2\pi)^{3/2}\Gamma(3N/2 - 3/2)} \frac{\int_0^\infty \Phi^2(\rho) \frac{1}{\rho^3} d\rho}{\int_0^\infty \Phi^2(\rho) d\rho} \ . \tag{37}$$

For large N, we use an approximate solution for $\Phi(\rho)$ [see Eq. (9)]:

$$\Phi(\rho) \approx \tilde{\rho}^{\frac{3N-1}{2}} e^{-(\tilde{\rho}/\alpha_t)^2/2} \tag{38}$$

where

$$\alpha_t = (\zeta/3)^{1/3} ,$$

$$\zeta \approx 2\sqrt{\frac{mc^2}{2\pi\hbar\omega}}\,\alpha N ,$$

and

$$\tilde{\rho} = \sqrt{\frac{m\omega}{\hbar}}\,\rho .$$

Using Eq. (38), we obtain from Eq. (37)

$$\tilde{R} = \frac{3AN}{8\pi\alpha}\sqrt{\frac{\hbar\omega}{mc^2}}\left(\frac{m\omega}{\hbar}\right)^{3/2} . \tag{39}$$

We can rewrite Eq. (39) as

$$\tilde{R} = BN\omega^2 , \tag{40}$$

where

$$B = \frac{3A}{8\pi\alpha}\left(\frac{m}{\hbar c}\right) . \tag{41}$$

If the probability of the ground-state occupation Ω is taken into account, the trap fusion rate is $R_t = \Omega\tilde{R}$.

We expect that this probability is very small for a low-density static case in the absence of an external potential.[8] However, Ω may not be very small for dynamic cases of Bose nuclei confined in ion traps in equilibrium situations.

Using Eq. (40), we have the following for the trap fusion rate R_t:

$$R_t = B\Omega N\omega^2 . \tag{42}$$

For the case of multiple ion traps in a metal with each trap containing N Bose nuclei, we define a trap number density n_t (number of traps per unit volume) as

$$n_t = \frac{N_t}{N} , \tag{43}$$

where N_t is the total number of Bose nuclei in traps per unit volume and N is the average number of Bose nuclei in a trap. For this case, the total ion trap nuclear fusion rate R per unit volume is

$$R = n_t R_t = n_t B\Omega N\omega^2 . \tag{44}$$

Both R_t and R do not depend on the Gamow factor in contrast to the conventional case. This is consistent with the conjecture noted by Dirac[4] and Bogolubov[5] that boson creation and annihilation operators can be treated simply as numbers when the ground-state occupation number is large. This implies that for large N, each charged boson behaves as an independent particle in a common average background potential, and the Coulomb interaction between two charged bosons is suppressed.

V. APPLICATION TO DEUTERON-DEUTERON FUSION

Using $S = 110$ keV·b for deuteron-deuteron fusion, we find from Eq. (35) the nuclear rate constant to be

$$A \approx 1.5 \times 10^{-16}\ \text{cm}^3/\text{s} , \tag{45}$$

and from Eqs. (41) and (45), we have

$$B = 2.6 \times 10^{-22}\ \text{s} . \tag{46}$$

For an ion trap containing $N \approx 10^6$ deuterons with $\omega \approx$ MHz $(=10^6\ \text{s}^{-1})$ (this may be achievable with the current experimental techniques), we find from Eq. (42) the following:

$$R_t = \Omega \times 2.6 \times 10^{-4}\ \text{s}^{-1} .$$

If Ω can be made larger $(\Omega \approx 10^{-2})$ by cooling the confined deuterons using currently available cooling techniques (laser cooling, collisional cooling, sympathetic cooling, or Sisyphus cooling[1]), we have $R_t \approx 10^{-6}\ \text{s}^{-1}$ (1/several days), which may be a measurable rate for protons and neutrons emitted from the reactions $d(d,p)t$ and $d(d,n)^3\text{He}$, respectively.

Recently, Yuki et al.[9] observed anomalous enhancement of the reaction $d(d,p)t$ with a Pd target and Au/Pd/PdO heterostructure target bombarded with low-energy deuteron beams ($E_d \approx 2.5$ to 10 keV laboratory energy), while other targets (Ti and Yb) do not yield anomalous enhancements of the fusion rate. Their data analysis with the conventional Gamow factor plus electron-screening energy U_s shows that anomalously large parametric values of $U_s \approx 250$ eV and $U_s \approx 600$ eV are required for the Pd and Au/Pd/PdO targets, respectively. These large values of U_s for the electron-screening energies cannot be explained by known conventional theories.

It is known that metal targets under ion beam bombardment develop surface blisters, which may consist of many microtraps (approximately micrometre size or less). If we assume $\sqrt{\hbar/m\omega} \approx 10^{-4}$ cm (corresponding to $\omega \approx 1.6 \times 10^4\ \text{s}^{-1}$) and the trap number density of $n_t = N_t/N$ with $N_t \approx 5.4 \times 10^{+19}\ \text{cm}^{-3}$, we obtain from Eq. (44)

$$R = \Omega \times 10^7 \frac{1}{\text{cm}^3 \cdot \text{s}} . \tag{47}$$

If Ω turns out to be $(10^{-7}$ to $10^{-10})$, we have $R \approx (10^{-3}$ to $1.0)\ \text{cm}^{-3}\cdot\text{s}^{-1}$, which may be consistent with anomalous low-energy enhancement of the cross section for

$d(d,p)t$ recently observed by Yuki et al.[9] Therefore, it is desirable to determine N_t by examining the Pd and Au/Pd/PdO targets used by Yuki et al.[9] in order to obtain more definitive tests of our ion trap nuclear fusion mechanism.

VI. SUMMARY AND CONCLUSIONS

Using the recently developed theoretical method (ELTB method),[3] we have obtained an approximate ground-state solution of the many-body Schrödinger equation for a system of N identical charged bosons confined in an isotropic harmonic oscillator potential. The solution is expected to be accurate for large N (Ref. 3). The solution is used to obtain theoretical formulas for estimating the probabilities and rates of nuclear fusion for N identical Bose nuclei confined in an ion trap. Our results show that the Coulomb interaction between two charged bosons is suppressed for the large N case. This is consistent with the conjecture made by Dirac[4] and used by Bogolubov[5] that each interacting (charged) boson behaves as an independent particle in a common average background for the large N case. The fusion-rate formula is applied to deuteron-deuteron fusion rates for deuterons confined in an ion trap and also in microtraps.

For a single-ion-trap case, our theoretical estimates of the nuclear fusion rate may be observable if we can achieve experimentally the total number of confined deuterons of $\sim 10^6$ and the ground-state occupation probability of 10^{-2} and 10^{-3}. Unless innovative new techniques are developed, it may be difficult to achieve the foregoing conditions by the use of currently available trapping and cooling techniques.

For the case of microtrap nuclear fusion in metals, it is desirable to test the microtrap nuclear fusion mechanism for recently observed anomalous increases in the fusion rate of $d(d,p)t$ when Pd and Au/Pd/PdO heterostructure targets are bombarded with low-energy deuteron beams.[9] The foregoing test can be done by measuring the number density of microtraps in various targets. If a correlation is found between the anomalous enhancement of the fusion rate and increased number density of microtraps in a target, then the microtrap nuclear fusion mechanism can be a plausible explanation for the observed anomalous effect. More direct experimental tests (beam or other types) should be carried out for more definitive conclusions. The same mechanism may also provide an explanation and a theoretical justification of the anomalous effect observed for deuteron-deuteron fusion by Arata and Zhang.[10,11]

REFERENCES

1. P. K. GHOSH, *Ion Traps*, Clarendon Press, Oxford, United Kingdom (1995).

2. Y. E. KIM and A. L. ZUBAREV, "Stability of the Solution of the Nonlinear Schrödinger Equation for the Bose-Einstein Condensation," *Phys. Lett. A*, **246**, 389 (1998).

3. Y. E. KIM and A. L. ZUBAREV, "Equivalent Linear Two-Body Method for Many-Body Problems," PNMBTG-99-2, Purdue Nuclear and Many-Body Theory Group (May 1999); see also *J. Physics B*: At. Mol. Opt. Phys. 33, 55 (2000).

4. P. A. M. DIRAC, *The Principles of Quantum Mechanics*, 2nd ed., Chap. XI, Sec. 62, Clarendon Press, Oxford, United Kingdom (1935).

5. N. BOGOLUBOV, "On the Theory of Superfluidity," *J. Phys.*, **11**, 23 (1947).

6. R. NEWTON, *Scattering Theory of Waves and Particles*, McGraw-Hill Book Company, New York (1966).

7. Y. E. KIM and A. L. ZUBAREV, "Optical Theorem and Finite-Range Effect for Nuclear Reactions in Astrophysics," *Few-Body Systems Suppl.*, **8**, 332 (1995).

8. L. L. FOLDY, "Charged Boson Gas," *Phys. Rev.*, **124**, 649 (1961).

9. H. YUKI, J. KASAGI, A. G. GIBSON, T. OHTSUKI, T. BABA, and T. NODA, "Anomalous Enhancement of DD Reaction in Pd and Au/Pd/PdO Heterostructure Targets Under Low-Energy Deuteron Bombardment," *JETP Lett.*, **68**, 823 (1998).

10. Y. ARATA and Y.-C. ZHANG, "Anomalous Difference Between Reaction Energies Generated Within D_2O-Cell and H_2O-Cell," *Jpn. J. Appl. Phys. B*, **7**, L1278 (1998).

11. Y. ARATA and Y.-C. ZHANG, "Observation of Anomalous Heat Release and Helium-4 Production from Highly Deuterated Palladium Fine Particles," *Jpn. J. Appl. Phys.*, **38**, L774 (1999).

Yeong E. Kim (PhD, University of California, Berkeley, 1963) is a professor of physics in the Department of Physics at Purdue University. In addition, he is group leader of the Purdue Nuclear and Many-Body Theory Group and director of the Division of Sponsored Program Development, Purdue Research Foundation, at Purdue University. His current research interests are in areas of nuclear

physics (nuclear astrophysics, nuclear fusion, compact high-flux neutron generator), condensed matter physics (Bose-Einstein condensation, nonlinear dynamics, quantum dots), and general relativity (black holes, Hawking radiation).

Alexander L. Zubarev (PhD, 1971, and Doctor of Science (Doctor of Habilitation), 1981, physics and mathematics, Joint Institute for Nuclear Research, Dubna, Russia) is a senior scientist in the Physics Department at Purdue University. He is well known for his research on the few-body problem in nuclear and atomic physics. His current research interests are few-body and many-body problems in nuclear and atomic physics and Bose-Einstein condensation.